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Controllability of Sobolev-Type Neutral Stochastic Mixed Integrodifferential Systems

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Abstract. This paper focuses on the controllability of Sobolev-type neutral stochastic mixed Volterra-Fredholm integrodifferential systems with nonlocal conditions in Hilbert spaces. Sufficient conditions for controllability are obtained by using resolvent operator and fixed point technique.

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1. Introduction

Differential equations play a central role in applications of mathematics to natural and engineering sciences. In general, more realistic formulation of the differential equations arising in applied sciences (taking into account uncertainities and random noises associated with the process considered) should involve stochastic differential equations. Recently, stochastic differential equations have been widely accepted as an important mathematical tool in modelling and analysis of numerous processes in engineering, especially in control and mechanical systems. Today the theory of stochastic differential equation has a very extensive literature dealing with both mathematical bases as well as applications.

Integrodifferential equations form a very rich class of equations. The study of integrodifferential equations is relatively a new area in mathematics full of open problems that attracts an increasing level of interest. Differential and integrodifferential equations, especially nonlinear, present the most effective way for describing complex processes. Most of the practical systems are integrodifferential equations in nature and hence the study of integrodifferential equations is very important. For instance, consider the longitudinal motion of a homogeneous one dimensional body in viscoelasticity.

$$\rho u_{tt}(t,\rho)+\beta u_t(t,\rho) \quad = \quad \triangle u(t,\rho)+\int_0^t F(t-s)\triangle u(s,\rho)ds+f(t,\rho),\ t\geq 0,$$

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$$u(0,\rho) = u_0(\rho), \ u_t(0,\rho) = u_1(\rho). \tag{1}$$

in $L^2(\Omega)$, where u is the displacement, ρ is the density per unit area and β is the coefficient of viscosity of the medium. As in [17], when density $\rho \to 0$, solutions of (1) will converge to solutions of the limiting heat equation

$$\beta Z_t(t) = \triangle Z(t) + \int_0^t F(t-s)\triangle Z(s)ds + f(t), \ t \geq 0, \ \ Z(0) = Z_0. \tag{2}$$

The associated abstract integrodifferential equation is given by

$$\frac{dz}{dt} = A\Big[z(t) + \int_0^t F(t-s)z(s)ds\Big] + f(t), \ t \geq 0, \ \ z(0) = z_0, \tag{3}$$

where A is a linear operator in a Hilbert space H and f is a real function. Practically, many physical and biological models are represented by a class mixed Volterra-Fredholm integrodifferential equations. The nonlinear mixed Volterra-Fredholm integrodifferential equations serve as models for various partial differential equations and partial integrodifferential equations arising in heat flow in material with memory, viscoelasticity and reaction diffusion problems. Some authors have studied the existence, uniqueness and controllability of such nonlinear mixed integrodifferential equations in Banach spaces [19, 30]. Now, we can generalize the above abstract form by imposing nonlinearity on f as used in [19, 30].

$$\begin{aligned}\frac{dz}{dt} &= A\Big[z(t) + \int_0^t F(t-s)z(s)ds\Big] \\ &\quad + f\Big(t, x(t), \int_0^t k\big(t,s,x(s)\big)ds, \int_0^a h\big(t,s,x(s)\big)ds\Big), \ t \geq 0, \ \ z(0) = z_0.\end{aligned} \tag{4}$$

Grimmer [12] and Oka [20] obtained the representation of solutions of integrodifferential equations (3) by using resolvent operators in Banach space. Ravikumar [22] derived the nonlocal cauchy problem for analytic resolvent integrodifferential equations in Banach spaces. Sakthivel et al. [25] discussed the existence and controllability result for semilinear evolution integrodifferential systems. Pazy [21] discussed the existence and uniqueness of mild, strong and classical solutions of semilinear evolution equations by employing semigroup method.

Sobolev-type equation appears in a variety of physical problems such as flow of fluid through fissured rocks [8], thermodynamics, propagation of long waves of small amplitude and shear in second order fluids and so on. Brill [2] and Showalter [24] established the existence of solutions of semilinear Sobolev type evolution equations in Banach space. There is an extensive literature in which Sobolev type of equations are investigated, in the abstract framework, see for instance [1, 4, 5, 7]. Controllability of Sobolev type integrodifferential systems in Banach spaces have been discussed in [3, 6]. Subsequently, Keck and Mckibben [14] derived a Mckean-Vlasov stochastic integrodifferential evolution equation of Sobolev type. Several authors have studied the existence and controllability of stochastic integrodifferential systems with and without impulsive conditions [9, 15, 18, 27, 29].

From the above literatures, it should be pointed out that there are several contributions on the existence and controllability of Sobolev type integrodifferential equations using semigroup method and the existence and controllability of integrodifferential equations with and without randomness using resolvent operators. Upto now, there is no work reported on the controllability of Sobolev-type neutral stochastic mixed integrodifferential system using resolvent operators. Motivated by this fact, in this paper, we make an attempt to fill this gap by studying the controllability of Sobolev-type neutral stochastic mixed integrodifferential systems with nonlocal conditions using resolvent operators.

2. Preliminaries

Consider the following class of Sobolev-type neutral stochastic mixed Volterra-Fredholm integrodifferential system with nonlocal conditions

$$
\begin{aligned}
d\Big[Bx(t)-g(t,x(t))\Big] &= A\Big[x(t)+\int_0^t F(t-s)x(s)ds\Big]dt + Cu(t)dt \\
&\quad + f\Big(t,x(t),\int_0^t k\big(t,s,x(s)\big)ds, \int_0^a h\big(t,s,x(s)\big)ds\Big)dt \\
&\quad + \sigma(t,x(t))dw(t), \quad t\in J:=[0,a], \\
x(0) &+ q(t_1,t_2,\cdots,t_p,x(t_1),x(t_2),\cdots,x(t_p)) = x_0, \qquad (5)
\end{aligned}
$$

where $0<t_1<t_2<\cdots<t_p\le a$ $(p\in N)$ and the state variable $x(\cdot)$ takes values in a real separable Hilbert space H with innerproduct $(\cdot,\cdot)$ and norm $\|\cdot\|$. Here A and B are linear operators on H and $F(t), t\in J$ is a bounded operator on H. The control function $u(\cdot)$ takes values in $L^2(J,U)$, a Banach space of admissible control functions for a separable Hilbert space U and C is a bounded linear operator from U into H. Let K be another separable Hilbert space with innerproduct $(\cdot,\cdot)_K$ and the norm $\|\cdot\|_K$. Suppose $\{w(t): t\ge 0\}$ is a given K-valued Wiener process with a finite trace nuclear covariance operator $Q\ge 0$. We employ the same notation $\|\cdot\|$ for the norm $\mathscr{L}(K,H)$, where $\mathscr{L}(K,H)$ denotes the space of all bounded linear operators from K into H. Further, assume that $g: J\times H\to H$, $f: J\times H\times H\times H\to H$, $k:\Delta\times H\to H$, $h:\Delta\times H\to H$, $\sigma: J\times H\ to\mathscr{L}_Q(K,H)$ are measurable mappings in H-norm and $\mathscr{L}_Q(K,H)$-norm respectively. Here $\mathscr{L}_Q(K,H)$ denotes the space of all Q-Hilbert-Schmidt operators from K into H which will be defined in Section 2 and $\Delta=\{(t,s)\in J\times J: s\le t\}$. The nonlocal function $q: C(J^p\times H^p,H)\to H$ is bounded and the initial data x_0 is an $\mathscr{F}_0$-adapted, H-valued random variable independent of Wiener process w.

Throughout the paper $(H,\|\cdot\|)$ and $(K,\|\cdot\|_K)$ denote real separable Hilbert spaces. Let $(\Omega,\mathscr{F},P;\mathbf{F})$ $\{\mathbf{F}=\{\mathscr{F}_t\}_{t\ge 0}\}$ be a complete filtered probability space satisfying that $\mathscr{F}_0$ contains all P-null sets of $\mathscr{F}$. An H-valued random variable is an $\mathscr{F}$-measurable function $x(t)$: $\Omega\to H$ and the collection of random variables $S=\{x(t,\omega):\Omega\to H \backslash t\in J\}$ is called a stochastic process. Generally, we just write $x(t)$ instead of $x(t,\omega)$ and $x(t): J\to H$ in the space of S. Let $\{e_i\}_{i=1}^{\infty}$ be a complete orthonormal basis of K. Suppose that $\{w(t): t\ge 0\}$ is a cylindrical K-valued wiener process with a finite trace nuclear covariance operator $Q\ge 0$, denote

$Tr(Q)=\sum_{i=1}^{\infty}\lambda_i=\lambda<\infty$, which satisfies that $Qe_i=\lambda_i e_i$. So, actually, $w(t)=\sum_{i=1}^{\infty}\sqrt{\lambda_i}\omega_i(t)e_i$, where $\{\omega_i(t)\}_{i=1}^{\infty}$ are mutually independent one-dimensional standard Wiener processes. We assume that $\mathscr{F}_t=\sigma\{w(s):0\le s\le t\}$ is the σ-algebra generated by w and $\mathscr{F}_a=\mathscr{F}$. Let $\Psi\in\mathscr{L}(K,H)$ and define

$$\|\Psi\|_Q^2=Tr(\Psi Q\Psi^*)=\sum_{n=1}^{\infty}\|\sqrt{\lambda_n}\Psi e_n\|^2.$$

If $\|\Psi\|_Q<\infty$, then Ψ is called a Q-Hilbert-Schmidt operator. Let $\mathscr{L}_Q(K,H)$ denote the space of all Q-Hilbert-Schmidt operators $\Psi:K\to H$. The completion $\mathscr{L}_Q(K,H)$ of $\mathscr{L}(K,H)$ with respect to the topology induced by the norm $\|\cdot\|_Q$ where $\|\Psi\|_Q^2=\langle\Psi,\Psi\rangle$ is a Hilbert space with the above norm topology. For more details refer to Da Prato [10].

The collection of all strongly measurable, square integrable H-valued random variables denoted by $\mathscr{L}_2(\Omega,\mathscr{F},P;H)\equiv\mathscr{L}_2(\Omega,H)$, is a Banach space equipped with norm $\|x(\cdot)\|_{\mathscr{L}_2}=(E\|x(\cdot;\omega)\|_H^2)^{\frac{1}{2}}$, where the expectation E is defined by $E(h)=\int_\Omega h(\omega)dP$. Similarly, $\mathscr{L}_2^{\mathscr{F}}(\Omega,H)$ denotes the Banach space of all $\mathscr{F}_t$-measurable, square integrable random variables, such that $\int_\Omega\|x(t,\cdot)\|_{\mathscr{L}_2}^2dt<\infty$. $\mathscr{C}(J,\mathscr{L}_2(\Omega,H))$ is the Banach space of all continuous maps from J into $\mathscr{L}_2(\Omega,H)$ satisfying the condition $\sup_{t\in J}E\|x(t)\|^2<\infty$. Let $\mathscr{C}(J,\mathscr{L}_2)$ be the closed subspace of all continuous process x that belong to the space $\mathscr{C}(J,\mathscr{L}_2^{\mathscr{F}}(\Omega,H))$ consisting of $\mathscr{F}_t$-adapted measurable processes $x(t)$. Then $\mathscr{C}(J,\mathscr{L}_2)$ is a Banach space endowed with the norm

$$\|x\|^2=\sup_{t\in J}\{E\|x(t)\|^2:x\in\mathscr{C}(J,\mathscr{L}_2)\}.$$

Now we consider the system (5) where the operators $A:D(A)\subset H\to H$ and $B:D(B)\subset H\to H$ satisfy the following hypotheses:

(E1) A and B are closed linear operators,

(E2) $D(B)\subset D(A)$ and B is bijective,

(E3) $B^{-1}:H\to D(B)$ is continuous.

Here, $(E1)$ and $(E2)$ together with the Closed Graph Theorem imply the boundedness of the linear operator $AB^{-1}:H\to H$. Further, AB^{-1} generates a strongly continuous semigroup of bounded linear operators in H. Let us denote $\|B^{-1}\|^2=M_B$ and $\|B\|^2=\tilde{M}_B$.

Here we recall some basic facts about resolvent operators and additional assumptions:

(E4) AB^{-1} generates a strongly continuous semigroup on H.

(E5) $F(t)\in B(H)$, $t\in J$. Also, $F(t):Y\to Y$ and for $x(\cdot)$ continuous in Y, $AF(\cdot)x(\cdot)\in L^1([0,a],H)$. For $x\in H$, $F'(t)x$ is continuous in $t\in J$, where $B(H)$ is the space of all bounded linear operators on H and Y is the Banach space formed from $D(A)$, the domain of A, endowed with the graph norm and $AB^{-1}F=FAB^{-1}$.

Definition 1. *A family of bounded linear operators $R(t) \in B(H)$ for $t \in J$ is called a resolvent operator for*

$$\frac{dx}{dt} = A\Big[x(t) + \int_0^t F(t-s)x(s)ds\Big] \tag{6}$$

if

(i) $R(0) = I$ *(the identity operator on H),*

(ii) for all $x \in H$, $R(t)x$ *is continuous for* $t \in J$,

(iii) $R(t) \in B(Y)$, $t \in J$. *For* $y \in Y$, $R(t)y \in C^1([0,a],H) \cap C([0,a],Y)$ *and*

$$\begin{aligned}\frac{d}{dt}R(t)y &= AB^{-1}\Big[R(t)y + \int_0^t F(t-s)R(s)yds\Big] \\ &= R(t)AB^{-1}y + \int_0^t R(t-s)AB^{-1}F(s)yds, \ t \in J.\end{aligned}$$

Here $R(t)$ *can be extracted from the semigroup operator of the generator* AB^{-1}.

Remark 1. *If* $F = 0$ *in the system* (5), *then the resolvent and the semigroup operator of the system generated by* AB^{-1} *are the same.*

Definition 2. *[11] A stochastic process x is said to be a mild solution of* (5) *if the following conditions are satisfied:*

(a) $x(t,\omega)$ *is a measurable function from* $J \times \Omega$ *to H and* $x(t)$ *is* $\mathscr{F}_t$ *-adapted for all* $t \in J$,

(b) $E\|x(t)\|^2 < \infty$ *for each* $t \in J$,

(c) For each $s \in [0,t)$ *the function* $B^{-1}R(t-s)AB^{-1}g(s,x(s))$ *is integrable and each* $u \in L_2^{\mathscr{F}}(J,U)$ *the process x satisfies the following integral equation:*

$$\begin{aligned}x(t) &= B^{-1}R(t)[Bx_0 - Bq\big(t_1,\cdots,t_p,x(t_1),\cdots,x(t_p)\big) - g(0,x(0))] + B^{-1}g(t,x(t)) \\ &\quad + \int_0^t B^{-1}R(t-s)Cu(s)ds + \int_0^t B^{-1}R(t-s)AB^{-1}g(s,x(s))ds \\ &\quad + \int_0^t B^{-1}R(t-s)AB^{-1}\Big[\int_0^s F(s-\tau)g(\tau,x(\tau))d\tau\Big]ds \\ &\quad + \int_0^t B^{-1}R(t-s)f\Big(s,x(s),\int_0^s k\big(s,\tau,x(\tau)\big)d\tau,\int_0^a h\big(s,\tau,x(\tau)\big)d\tau\Big)ds \\ &\quad + \int_0^t B^{-1}R(t-s)\sigma(s,x(s))dw(s) \ \text{ for a.e. } t \in J, \\ x(0) &+ q\big(t_1,t_2,\cdots,t_p,x(t_1),x(t_2),\cdots,x(t_p)\big) = x_0 \in H.\end{aligned} \tag{7}$$

Definition 3. *The system* (5) *is said to be controllable on the interval J, if for every initial condition x_0 and $x_1 \in H$, there exists a control $u \in L^2(J,U)$ such that the solution $x(\cdot)$ of* (5) *satisfies $x(a) = x_1$.*

It is known from the properties of resolvent operator discussed in [12], there exists a constant $M \geq 1$ and a real number w such that

$$\|R(t)\| \leq Me^{wt}, \ t \in J.$$

We assume that $\|R(t)\|$ is uniformly bounded by M. Before proceeding to the main result we shall set forth a list of hypotheses:

(H1) AB^{-1} is the infinitesimal generator of a resolvent operator $R(t)$ in H and there exist constants $M > 0$ and $M_F > 0$ such that

$$\|R(t)\|^2 \leq M, \quad \|F(t)\|^2 \leq M_F, \quad \text{for all } t \in J.$$

(H2) The linear operator $W : L^2(J,U) \to H$ defined by

$$Wu = \int_0^a B^{-1}R(a-s)Cu(s)ds$$

is invertible with inverse operator W^{-1} taking values in $L^2(J,U) \setminus kerW$ and there exist positive constants M_C, M_W such that

$$\|C\|^2 \leq M_C, \ \|W^{-1}\|^2 \leq M_W.$$

(H3) (i) The nonlinear function $g : J \times H \to H$ is continuous and there exist constants $M_g > 0$, $\tilde{M}_g > 0$ for $t,s \in J$ and $x,y \in H$ such that the function $AB^{-1}g$ satisfies the Lipschitz condition:

$$E\|AB^{-1}g(t,x) - AB^{-1}g(t,y)\|^2 \leq M_g\|x-y\|^2$$

and $\tilde{M}_g = \sup_{t\in J}\|AB^{-1}g(t,0)\|^2$.

(ii) There exist constants $M_1 > 0, M_2 > 0$ and $M_3 > 0$ such that

$$\begin{aligned} E\|g(t,x)-g(s,y)\|^2 &\leq M_1\left(|t-s|^2 + \|x-y\|^2\right) \text{ and} \\ E\|g(t,x)\|^2 &\leq M_2\|x\|^2 + M_3, \end{aligned}$$

where $M_3 = \sup_{t\in J}\|g(t,0)\|^2$.

(H4) The nonlinear function $f : J \times H \times H \times H \to H$ is continuous and there exist constants $M_f > 0$, $\tilde{M}_f > 0$ for $t \in J$ and $x_1, x_2, y_1, y_2, z_1, z_2 \in H$ such that

$$E\|f(t,x_1,y_1,z_1) - f(t,x_2,y_2,z_2)\|^2 \leq M_f\left[\|x_1-x_2\|^2 + \|y_1-y_2\|^2 + \|z_1-z_2\|^2\right]$$

and $\tilde{M}_f = \sup_{t\in J}\|f(t,0,0,0)\|^2$.

(H5) The function $k : \Delta \times H \to H$ is continuous and there exist positive constants M_k, $\tilde{M}_k$, for $x, y \in H$ and $(t,s) \in \Delta$ such that

$$E\left\|\int_0^t \Big(k(t,s,x) - k(t,s,y)\Big) ds\right\|^2 \le M_k \|x-y\|^2$$

and $\tilde{M}_k = \sup_{(t,s)\in\Delta} \|\int_0^t k(t,s,0)ds\|^2$.

(H6) The function $h : \Delta \times H \to H$ is continuous and there exist positive constants M_h, $\tilde{M}_h$, for $x, y \in H$ and $(t,s) \in \Delta$ such that

$$E\left\|\int_0^a \Big(h(t,s,x) - h(t,s,y)\Big) ds\right\|^2 \le M_h \|x-y\|^2$$

and $\tilde{M}_h = \sup_{(t,s)\in\Delta} \|\int_0^a h(t,s,0)ds\|^2$.

(H7) The function $\sigma : J \times H \to \mathscr{L}_Q(K,H)$ is continuous and there exist constants $M_\sigma > 0$, $\tilde{M}_\sigma > 0$ for $t \in J$ and $x, y \in H$ such that

$$E\|\sigma(t,x) - \sigma(t,y)\|_Q^2 \le M_\sigma \|x-y\|^2$$

and $\tilde{M}_\sigma = \sup_{t\in J} \|\sigma(t,0)\|^2$.

(H8) The nonlocal function $q : C(J^p \times H^p, H) \to H$ is continuous and there exist constants $M_q > 0$, $\tilde{M}_q > 0$ for $x, y \in H$ such that

$$E\|q(t_1, \cdots, t_p, x(t_1), \cdots, x(t_p)) - q(t_1, \cdots, t_p, y(t_1), \cdots, y(t_p))\|^2 \le M_q \|x-y\|^2,$$

$$E\|q(t_1, t_2, \cdots, t_p, x(t_1), x(t_2), \cdots, x(t_p))\|^2 \le \tilde{M}_q.$$

(H9) There exists a constant $r > 0$ such that

$$\begin{aligned}
9\Big\{&M_B M\ \tilde{M}_B\ (\|x_0\|^2 + \tilde{M}_q) + 2M_B M\left[M_2(\|x_0\|^2 + \tilde{M}_q) + M_3\right] + M_B[M_2 r + M_3] \\
&+ 2a^2 M_B M(1+aM_F)[M_g r + \tilde{M}_g] + a^2 M_B M M_C \mathscr{G} + 2aM_B M Tr(Q)[M_\sigma r \\
&+ \tilde{M}_\sigma] + 2a^2 M_B M\left[M_f\left((1+2M_k+2M_h)r + 2(\tilde{M}_k + \tilde{M}_h)\right) + \tilde{M}_f\right]\Big\} \le r
\end{aligned}$$

$$\begin{aligned}
\text{and } \delta &= 8\{(1 + 7a^2 M_B M M_C M_W)(N_1 + N_2 + N_3 + N_4 + N_5)\}, \\
\text{where } N_1 &= M_B M \tilde{M}_B M_q, \\
N_2 &= M_B M M_1 M_q + M_B M_1, \\
N_3 &= a^2 M_B M M_g (1 + aM_F), \\
N_4 &= a^2 M_B M M_f (1 + M_k + M_h), \\
N_5 &= aM_B M Tr(Q) M_\sigma.
\end{aligned}$$

3. Controllability Result

Theorem 1. *If the conditions* $(H1)-(H9)$ *are satisfied and if* $0 \le \delta < 1$*, then the system* (5) *is controllable on* J.

Proof: Using the hypothesis $(H2)$ for an arbitrary function $x(\cdot)$, define the control

$$\begin{aligned} u(t) = W^{-1}\Big[& x_1 - B^{-1}R(a)\big[Bx_0 - Bq\big(t_1,\cdots,t_p,x(t_1),\cdots,x(t_p)\big) - g(0,x(0))\big] - B^{-1}g(a,x(a)) \\ & - \int_0^a B^{-1}R(a-s)AB^{-1}g(s,x(s))ds - \int_0^a B^{-1}R(a-s)AB^{-1}\Big[\int_0^s F(s-\tau)g(\tau,x(\tau))d\tau\Big]ds \\ & - \int_0^a B^{-1}R(a-s)f\Big(s,x(s),\int_0^s k\big(s,\tau,x(\tau)\big)d\tau,\int_0^a h\big(s,\tau,x(\tau)\big)d\tau\Big)ds \\ & - \int_0^a B^{-1}R(a-s)\sigma(s,x(s))dw(s)\Big](t). \end{aligned} \tag{8}$$

Let $\mathscr{Y}_r$ be a nonempty closed subset of $\mathscr{C}(J,\mathscr{L}_2)$ defined by

$$\mathscr{Y}_r = \{x : x \in \mathscr{C}(J,\mathscr{L}_2) | E\|x(t)\|^2 \le r\}.$$

Consider a mapping $\Psi : \mathscr{Y}_r \to \mathscr{Y}_r$ defined by

$$\begin{aligned} (\Psi x)(t) \;=\; & B^{-1}R(t)\big[Bx_0 - Bq\big(t_1,\cdots,t_p,x(t_1),\cdots,x(t_p)\big) - g(0,x(0))\big] + B^{-1}g(t,x(t)) \\ & + \int_0^t B^{-1}R(t-s)Cu(s)ds + \int_0^t B^{-1}R(t-s)AB^{-1}g(s,x(s))ds \\ & + \int_0^t B^{-1}R(t-s)AB^{-1}\Big[\int_0^s F(s-\tau)g(\tau,x(\tau))d\tau\Big]ds \\ & + \int_0^t B^{-1}R(t-s)f\Big(s,x(s),\int_0^s k\big(s,\tau,x(\tau)\big)d\tau,\int_0^a h\big(s,\tau,x(\tau)\big)d\tau\Big)ds \\ & + \int_0^t B^{-1}R(t-s)\sigma(s,x(s))dw(s). \end{aligned}$$

Since all the functions involved in the operator are continuous therefore Ψ is continuous. From our assumptions we have

$$\begin{aligned} E\|u_x(t)\|^2 \le 9M_W\Big\{ & \|x_1\|^2 + M_BM\tilde{M}_B\|x_0\|^2 + M_BM\tilde{M}_B\tilde{M}_q + 2M_BM\left[M_2(\|x_0\|^2 + \tilde{M}_q) + M_3\right] \\ & + M_B(M_2r + M_3) + 2a^2M_BM(1 + aM_F)[M_gr + \tilde{M}_g] + 2aM_BMTr(Q)[M_\sigma r \\ & + \tilde{M}_\sigma] + 2a^2M_BM\left[M_f\left((1 + 2M_k + 2M_h)r + 2(\tilde{M}_k + \tilde{M}_h)\right) + \tilde{M}_f\right]\Big\} := \mathscr{G} \end{aligned}$$

and

$$E\|u_x(t) - u_y(t)\|^2 \;\le\; 7M_W\Big\{M_BM\tilde{M}_BM_q + M_BMM_1M_q + M_BM_1 + a^2M_BMM_g(1 + aM_F)$$

$$+a^2M_BMM_f(1+M_k+M_h)+aM_BMTr(Q)M_\sigma\Big\}\|x-y\|^2.$$

We shall show that by using the above control the operator Ψ has a fixed point.
Step 1: The operator Ψ maps $\mathscr{Y}_r$ into $\mathscr{Y}_r$.

$$\begin{aligned}
E\|\Psi x(t)\|^2 \leq\ & 9\Big\{E\|B^{-1}R(t)\big[Bx_0-Bq\big(t_1,\cdots,t_p,x(t_1),\cdots,x(t_p)\big)-g(0,x(0))\big]\|^2 \\
& +E\|B^{-1}g(t,x(t))\|^2+E\Big\|\int_0^t B^{-1}R(t-s)AB^{-1}g(s,x(s))ds\Big\|^2 \\
& +E\Big\|\int_0^t B^{-1}R(t-s)AB^{-1}\Big[\int_0^s F(s-\tau)g(\tau,x(\tau))d\tau\Big]ds\Big\|^2 \\
& +E\Big\|\int_0^t B^{-1}R(t-s)f\Big(s,x(s),\int_0^s k\big(s,\tau,x(\tau)\big)d\tau,\int_0^a h\big(s,\tau,x(\tau)\big)d\tau\Big)ds\Big\|^2 \\
& +E\Big\|\int_0^t B^{-1}R(t-s)Cu(s)ds\Big\|^2+E\Big\|\int_0^t B^{-1}R(t-s)\sigma(s,x(s))dw(s)\Big\|^2\Big\} \\
\leq\ & 9\Big\{M_BM\tilde{M}_B\|x_0\|^2+M_BM\tilde{M}_B\tilde{M}_q+2M_BM\big[M_2(\|x_0\|^2+\tilde{M}_q)+M_3\big]+M_B[M_2r \\
& +M_3]+2a^2M_BM(1+aM_F)[M_gr+\tilde{M}_g]+a^2M_BMM_C\mathscr{G}+2aM_BMTr(Q)\times \\
& \times[M_\sigma r+\tilde{M}_\sigma]+2a^2M_BM\big[M_f\big((1+2M_k+2M_h)r+2(\tilde{M}_k+\tilde{M}_h)\big)+\tilde{M}_f\big]\Big\}.
\end{aligned}$$

From $(H9)$ we obtain $E\|(\Psi x)(t)\|^2\leq r$. Hence Ψ maps $\mathscr{Y}_r$ into itself.
Step 2: $\Psi:\mathscr{Y}_r\to\mathscr{Y}_r$ is a contraction mapping.
Let $x,y\in\mathscr{Y}_r$, then

$$\begin{aligned}
& E\|(\Psi x)(t)-(\Psi y)(t)\|^2 \\
& \leq 8\Big\{E\Big\|B^{-1}R(t)\big[Bx_0-g(0,x(0))-Bx_0+g(0,y(0))\big]\Big\|^2 \\
& +E\Big\|B^{-1}R(t)[-Bq\big(t_1,\cdots,t_p,x(t_1),\cdots,x(t_p)\big)+Bq\big(t_1,\cdots,t_p,y(t_1),\cdots,y(t_p)\big)]\Big\|^2 \\
& +E\Big\|B^{-1}\big[g(t,x(t))-g(t,y(t))\big]\Big\|^2+E\Big\|\int_0^t B^{-1}R(t-s)C\big[u_x(s)-u_y(s)\big]ds\Big\|^2 \\
& +E\Big\|\int_0^t B^{-1}R(t-s)AB^{-1}\big[g(s,x(s))-g(s,y(s))\big]ds\Big\|^2 \\
& +E\Big\|\int_0^t B^{-1}R(t-s)AB^{-1}\Big[\int_0^s F(s-\tau)\big[g(\tau,x(\tau))-g(\tau,y(\tau))\big]d\tau\Big]ds\Big\|^2 \\
& +E\Big\|\int_0^t B^{-1}R(t-s)\big[f\big(s,x(s),\int_0^s k\big(s,\tau,x(\tau)\big)d\tau,\int_0^a h\big(s,\tau,x(\tau)\big)d\tau\big) \\
& \qquad -f\big(s,y(s),\int_0^s k\big(s,\tau,y(\tau)\big)d\tau,\int_0^a h\big(s,\tau,y(\tau)\big)d\tau\big)\big]ds\Big\|^2
\end{aligned}$$

$$
\begin{aligned}
&+E\Big\|\int_0^t B^{-1}R(t-s)\big[\sigma(s,x(s))-\sigma(s,y(s))\big]dw(s)\Big\|^2\Big\}\\
&\leq 8\big\{(1+7a^2M_BMM_CM_W)\big[M_BM\tilde{M}_BM_q+M_BMM_1M_q+a^2M_BMM_g(1+aM_F)\\
&\qquad +M_BM_1+a^2M_BMM_f(1+M_k+M_h)+aM_BMTr(Q)M_\sigma\big]\big\}\|x-y\|^2\\
&\leq 8\big\{(1+7a^2M_BMM_CM_W)(N_1+N_2+N_3+N_4+N_5)\big\}\|x-y\|^2\\
&\leq \delta\|x-y\|^2.
\end{aligned}
$$

Since $\delta<1$, the mapping Ψ is a contraction and hence by Banach fixed point theorem there exists a unique fixed point $x\in\mathscr{Y}_r$ such that $(\Psi x)(t)=x(t)$. This fixed point is then the solution of the system (5) and clearly, $x(a)=(\Psi x)(a)=x_1$ which implies that the system (5) is controllable on J.

Remark 2. *Consider the following class of Sobolev-type neutral integrodifferential system*

$$
\begin{aligned}
d\Big[Bx(t)-Q\Big(t,x(t),\int_0^t q(t,s,x(s))ds\Big)\Big] &= A\Big[x(t)+\int_0^t F(t-s)x(s)ds\Big]dt+Cu(t)dt\\
&\quad +f\Big(t,x(t),\int_0^t k\big(t,s,x(s)\big)ds,\int_0^a h\big(t,s,x(s)\big)ds\Big)dt\\
&\quad +G\Big(t,x(t),\int_0^t \sigma(t,s,x(s))ds\Big)dw(t),\, t\in J,\\
x(0)+g(x) &= x_0, \qquad (9)
\end{aligned}
$$

where A,B,C,F,f,k,h are defined as before. Further,

$$
\begin{aligned}
&Q:J\times H\times H\to H, \qquad G:J\times H\times H\to \mathscr{L}_Q(K,H),\\
&q:\Delta\times H\to H, \qquad \sigma:\Delta\times H\to H,\ g:C(J,H)\to H
\end{aligned}
$$

are measurable mappings in H-norm and $\mathscr{L}_Q(K,H)$-norm respectively. The solution of the above equation is

$$
\begin{aligned}
x(t) &= B^{-1}R(t)[Bx_0-Bg(x)-Q(0,x(0),0)]+B^{-1}Q\Big(t,x(t),\int_0^t q(t,s,x(s))ds\Big)\\
&\quad +\int_0^t B^{-1}R(t-s)Cu(s)ds+\int_0^t B^{-1}R(t-s)AB^{-1}Q\Big(s,x(s),\int_0^s q(s,\eta,x(\eta))d\eta\Big)ds\\
&\quad +\int_0^t B^{-1}R(t-s)AB^{-1}\Big[\int_0^s F(s-\tau)Q\Big(\tau,x(\tau),\int_0^\tau q(\tau,\eta,x(\eta))d\eta\Big)d\tau\Big]ds\\
&\quad +\int_0^t B^{-1}R(t-s)f\Big(s,x(s),\int_0^s k\big(s,\tau,x(\tau)\big)d\tau,\int_0^a h\big(s,\tau,x(\tau)\big)d\tau\Big)ds\\
&\quad +\int_0^t B^{-1}R(t-s)G\Big(s,x(s),\int_0^s \sigma(s,\eta,x(\eta))d\eta\Big)dw(s)\ \text{ for a.e. } t\in J. \qquad (10)
\end{aligned}
$$

Concerning the operators Q,q,G,σ,g we assume the following hypotheses:

(H10) (i) The function $Q : J \times H \times H \to H$ is continuous and there exist constants $\mathscr{C}_Q > 0$, $\tilde{\mathscr{C}}_Q > 0$ for $t, s \in J$ and $x, y, x_1, y_1 \in H$ such that the function $AB^{-1}Q$ satisfies the Lipschitz condition:

$$E\|AB^{-1}[Q(t,x,x_1) - Q(t,y,y_1)]\|^2 \leq C_Q\big(\|x-y\|^2 + \|x_1 - y_1\|^2\big)$$

and $\tilde{\mathscr{C}}_Q = \sup_{t\in J} \|AB^{-1}Q(t,0,0)\|^2$.

(ii) There exist constants $Q_1 > 0, Q_2 > 0$ and $Q_3 > 0$ such that

$$\begin{aligned} E\|Q(t,x,x_1) - Q(s,y,y_1)\|^2 &\leq Q_1\big(|t-s|^2 + \|x-y\|^2 + \|x_1 - y_1\|^2\big) \text{ and} \\ E\|Q(t,x,y)\|^2 &\leq Q_2\big(\|x\|^2 + \|y\|^2\big) + Q_3, \end{aligned}$$

where $Q_3 = \sup_{t\in J} \|Q(t,0,0)\|^2$.

(H11) The nonlinear function $q : \Delta \times H \to H$ is continuous and there exist positive constants $\mathscr{C}_q$, $\tilde{\mathscr{C}}_q$, for $x, y \in H$ and $(t,s) \in \Delta$ such that

$$E\Big\| \int_0^t \big(q(t,s,x) - q(t,s,y)\big)\,ds \Big\|^2 \leq \mathscr{C}_q \|x-y\|^2$$

and $\tilde{\mathscr{C}}_q = \sup_{(t,s)\in\Delta} \|\int_0^t q(t,s,0)ds\|^2$.

(H12) The nonlinear function $G : J\times H\times H \to \mathscr{L}_Q(K,H)$ is continuous and there exist constants $\mathscr{C}_G > 0$, $\tilde{\mathscr{C}}_G > 0$ for $t \in J$ and $x_1, x_2, y_1, y_2 \in H$ such that

$$E\|G(t,x_1,y_1) - G(t,x_2,y_2)\|^2 \leq \mathscr{C}_G\big(\|x_1 - x_2\|^2 + \|y_1 - y_2\|^2\big)$$

and $\tilde{\mathscr{C}}_G = \sup_{t\in J} \|G(t,0,0)\|^2$.

(H13) The nonlinear function $\sigma : \Delta \times H \to H$ is continuous and there exist positive constants $\mathscr{C}_\sigma$, $\tilde{\mathscr{C}}_\sigma$, for $x, y \in H$ and $(t,s) \in \Delta$ such that

$$E\Big\| \int_0^t \big(\sigma(t,s,x) - \sigma(t,s,y)\big)\,ds \Big\|^2 \leq \mathscr{C}_\sigma \|x-y\|^2$$

and $\tilde{\mathscr{C}}_\sigma = \sup_{(t,s)\in\Delta} \|\int_0^t \sigma(t,s,0)ds\|^2$.

(H14) The nonlocal function $g : C(J,H) \to H$ is continuous and there exist constants $\mathscr{C}_g > 0$, $\tilde{\mathscr{C}}_g > 0$ for $x, y \in H$ such that

$$E\|g(x) - g(y)\|^2 \leq \mathscr{C}_g \|x-y\|^2, \quad E\|g(x)\|^2 \leq \tilde{\mathscr{C}}_g.$$

(H15) There exists a constant $\rho > 0$ such that

$$9\Big\{M_B M \tilde{M}_B\left(\|x_0\|^2 + \tilde{\mathscr{C}}_g\right) + 2M_B M\left[Q_2(\|x_0\|^2 + \tilde{\mathscr{C}}_g) + Q_3\right] + M_B[Q_2((1+2\mathscr{C}_q)\rho + 2\tilde{\mathscr{C}}_q)$$

$$
\begin{aligned}
& + Q_3] + 2a^2 M_B M(1+aM_F)[\mathscr{C}_Q((1+2\mathscr{C}_q)\rho + 2\tilde{\mathscr{C}}_q) + \tilde{\mathscr{C}}_Q] + a^2 M_B M M_C \mathscr{G} \\
& + 2a^2 M_B M \left[M_f \left((1+2M_k+2M_h)\rho + 2(\tilde{M}_k + \tilde{M}_h) \right) + \tilde{M}_f \right] \Big\} \\
& + 2aM_B M Tr(Q)[\mathscr{C}_G((1+2\mathscr{C}_\sigma)\rho + 2\tilde{\mathscr{C}}_\sigma) + \tilde{\mathscr{C}}_G] \le \rho \text{ and} \\
\delta^* \; & = 8\{(1+7a^2 M_B M M_C M_W)(L_1+L_2+L_3+L_4+L_5)\}, \\
\text{where } L_1 \; & = \; M_B M \tilde{M}_B \mathscr{C}_g, \\
L_2 \; & = \; M_B M Q_1 \mathscr{C}_g + M_B Q_1 (1+\mathscr{C}_q), \\
L_3 \; & = \; a^2 M_B M (1+aM_F)\mathscr{C}_Q(1+\mathscr{C}_q), \\
L_4 \; & = \; a^2 M_B M M_f (1+M_k+M_h), \\
L_5 \; & = \; aM_B M Tr(Q)\mathscr{C}_G(1+\mathscr{C}_\sigma).
\end{aligned}
$$

Let $\mathscr{Y}_\rho$ be defined by $\mathscr{Y}_\rho = \{x : x \in \mathscr{C}(J, \mathscr{L}_2) | E\|x(t)\|^2 \le \rho\}$ and the operator $\Phi : \mathscr{Y}_\rho \to \mathscr{Y}_\rho$ is defined as

$$
\begin{aligned}
\Phi x(t) \; = \; & B^{-1}R(t)[Bx_0 - Bg(x) - Q(0,x(0),0)] + B^{-1}Q\Big(t, x(t), \int_0^t q(t,s,x(s))ds\Big) \\
& + \int_0^t B^{-1}R(t-s)Cu(s)ds + \int_0^t B^{-1}R(t-s)AB^{-1}Q\Big(s, x(s), \int_0^s q(s,\eta,x(\eta))d\eta\Big)ds \\
& + \int_0^t B^{-1}R(t-s)AB^{-1}\Big[\int_0^s F(s-\tau)Q\Big(\tau, x(\tau), \int_0^\tau q(\tau,\eta,x(\eta))d\eta\Big)d\tau\Big]ds \\
& + \int_0^t B^{-1}R(t-s)f\Big(s, x(s), \int_0^s k\big(s,\tau,x(\tau)\big)d\tau, \int_0^a h\big(s,\tau,x(\tau)\big)d\tau\Big)ds \\
& + \int_0^t B^{-1}R(t-s)G\Big(s, x(s), \int_0^s \sigma(s,\eta,x(\eta))d\eta\Big)dw(s),
\end{aligned}
$$

where

$$
\begin{aligned}
u(t) = W^{-1}\Big[& x_1 - B^{-1}R(a)[Bx_0 - Bg(x) - Q(0,x(0),0)] - B^{-1}Q\Big(a, x(a), \int_0^a q(a,s,x(s))ds\Big) \\
& - \int_0^a B^{-1}R(a-s)AB^{-1}Q\Big(s, x(s), \int_0^s q(s,\eta,x(\eta))d\eta\Big)ds \\
& - \int_0^a B^{-1}R(a-s)AB^{-1}\Big[\int_0^s F(s-\tau)Q\Big(\tau, x(\tau), \int_0^\tau q(\tau,\eta,x(\eta))d\eta\Big)d\tau\Big]ds \\
& - \int_0^a B^{-1}R(a-s)f\Big(s, x(s), \int_0^s k\big(s,\tau,x(\tau)\big)d\tau, \int_0^a h\big(s,\tau,x(\tau)\big)d\tau\Big)ds \\
& - \int_0^a B^{-1}R(a-s)G\Big(s, x(s), \int_0^s \sigma(s,\eta,x(\eta))d\eta\Big)dw(s)\Big](t).
\end{aligned}
$$

Clearly the above control operator transfers the system (9) from the initial state x_0 to the final state x_1 provided that the operator Φx has a fixed point. Hence, if the operator Φx has a fixed point then the system (9) is controllable.

Theorem 2. *If the conditions* $(H1),(H2),(H4)-(H6),(H10)-(H15)$ *hold, then the system* (9) *is controllable provided*

$$8\{(1+7a^2M_BMM_CM_W)(L_1+L_2+L_3+L_4+L_5)\}<1.$$

Proof. The proof of this theorem is similar to that of Theorem 1 and hence it is omitted.

4. Sobolev-Type Stochastic Neutral Impulsive Systems

The theory of impulsive differential equations in the field of modern applied mathematics has made considerable headway in recent years, because the structure of its emergence has deep physical background and realistic mathematical models. For the basic theory on impulsive differential equations refer to [16, 23]. Karthikeyan and Balachandran [13] studied the controllability of nonlinear stochastic neutral impulsive systems. Sakthivel et al. [26] derived the controllability of nonlinear impulsive stochastic systems. Subalakshmi and Balachandran [28] discussed approximate controllability of nonlinear stochastic impulsive integrodifferential systems in Hilbert spaces. Moreover, the controllability of Sobolev type stochastic neutral impulsive systems is an untreated topic in the literature sofar. Motivated by this fact, in this section we study the controllability of Sobolev type stochastic neutral impulsive mixed integrodifferential systems with nonlocal conditions of the form

$$\begin{aligned}
d\left[Bx(t)-g(t,x(t))\right] = \Big[A[x(t)-g(t,x(t))]+f\Big(t,x(t),\int_0^t k\big(t,s,x(s)\big)ds,\int_0^a h\big(t,s,x(s)\big)ds\Big)\\
+Cu(t)\Big]dt+\sigma(t,x(t))dw(t),\quad t\neq t_k,\ t\in J:=[0,a],\\
\triangle x(t_k) \;=\; x(t_k^+)-x(t_k^-)=I_k(x(t_k^-)),\quad k=1,2,\cdots,m,\\
x(0)+H(x) \;=\; x_0, \qquad\qquad\qquad\qquad (11)
\end{aligned}$$

where A,B,C,f,g,k,h,σ are defined as in Section 2. Also, from Remark 2.1 we know that AB^{-1} is the infinitesimal generator of a strongly continuous semigroup $\mathscr{S}(t),\ t\geq 0$ in H. Here, the nonlocal function $H:\mathscr{P}C(J,H)\to H$ and impulsive function $I_k\in C(H,H)\ (k=1,2,\cdots,m)$ are bounded functions. Furthermore, the fixed times t_k satisfies $0=t_0<t_1<t_2<\cdots<t_m<a$, $x(t_k^+)$ and $x(t_k^-)$ denote the right and left limits of $x(t)$ at $t=t_k$. And $\triangle x(t_k)=x(t_k^+)-x(t_k^-)$ represents the jump in the state x at time t_k, where I_k determines the size of the jump. Denote $J_0=[0,t_1],J_k=(t_k,t_{k+1}],k=1,2,\cdots,m$, and define the following class of functions:

$\mathscr{P}C(J,\mathscr{L}_2(\Omega,H))=\{x:J\to\mathscr{L}_2:$ x(t) is continuous everywhere except for some t_k at which $x(t_k^-)$ and $x(t_k^+)$ exists and $x(t_k^-)=x(t_k),k=1,2,3,\cdots,m\}$ is the Banach space of piecewise continuous maps from J into $\mathscr{L}_2(\Omega,H)$ satisfying the condition $\sup_{t\in J}E\|x(t)\|^2<\infty$. Let $\mathscr{P}C(J,\mathscr{L}_2)$ be the closed subspace of $\mathscr{P}C(J,\mathscr{L}_2^{\mathscr{F}}(\Omega,H))$ consisting of measurable, $\mathscr{F}_t$-adapted and H-valued processes $x(t)$. Then $\mathscr{P}C(J,\mathscr{L}_2)$ is a Banach space endowed with the norm

$$\|x\|^2_{\mathscr{P}C} = \sup_{t\in J} \ \{E\|x(t)\|^2 : x \in \mathscr{P}C(J,\mathscr{L}_2)\}.$$

The solution of the above equation is given by

$$\begin{aligned} x(t) &= B^{-1}\mathscr{S}(t)[Bx_0 - BH(x) - g(0,x(0))] + B^{-1}g(t,x(t)) + \int_0^t B^{-1}\mathscr{S}(t-s)Cu(s)ds \\ &\quad + \int_0^t B^{-1}\mathscr{S}(t-s)f\Big(s,x(s),\int_0^s k\big(s,\tau,x(\tau)\big)d\tau, \int_0^a h\big(s,\tau,x(\tau)\big)d\tau\Big)ds \\ &\quad + \int_0^t B^{-1}\mathscr{S}(t-s)\sigma(s,x(s))dw(s) + \sum_{0<t_k<t} B^{-1}\mathscr{S}(t-t_k)I_k(x(t_k^-)), \text{for a.e. } t\in J, \\ \triangle x(t_k) &= x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), \quad k = 1,2,\cdots,m, \\ x(0) &+ H(x) = x_0 \in H. \end{aligned} \tag{12}$$

In order to prove the main result we shall assume some additional hypotheses:

(H16) AB^{-1} is the infinitesimal generator of a C_0 semigroup $\mathscr{S}(t)$ in H and there exists constant $\tilde{M} > 0$ such that

$$\|\mathscr{S}(t)\|^2 \le \tilde{M}, \text{ for all } t \in J.$$

(H17) The linear operator $\tilde{W} : L^2(J,U) \to H$ defined by

$$\tilde{W}u = \int_0^a B^{-1}\mathscr{S}(a-s)Cu(s)ds$$

is invertible with inverse operator $\tilde{W}^{-1}$ take values in $L^2(J,U) \setminus ker\tilde{W}$ and there exist positive constants $\tilde{M}_C$, $\tilde{M}_W$ such that

$$\|C\|^2 \le \tilde{M}_C, \ \|\tilde{W}^{-1}\|^2 \le \tilde{M}_W.$$

(H18) The nonlocal function $H : \mathscr{P}C(J,H) \to H$ is continuous and there exist constants $M_H > 0$, $\tilde{M}_H > 0$ such that

$$E\|H(x) - H(y)\|^2 \le M_H\|x-y\|^2, \ \ E\|H(x)\|^2 \le \tilde{M}_H.$$

(H19) $I_k : H \to H$ is continuous and there exist constants $\beta_k > 0$, $\tilde{\beta_k} > 0$ such that

$$E\|I_k(x) - I_k(y)\|^2 \le \beta_k\|x-y\|^2, \ \ k = 1,2,\cdots,m$$

and $\tilde{\beta_k} = \|I_k(0)\|^2$, $k = 1,2,\cdots,m$.

(H20) There exists a constant $r^* > 0$ such that

$$
\begin{aligned}
8\Big\{ & M_B\tilde{M}\ \tilde{M}_B\ (\|x_0\|^2+\tilde{M}_H)+2M_B\tilde{M}\left[M_2(\|x_0\|^2+\tilde{M}_H)+M_3\right]+M_B[M_2r^*+M_3] \\
& +a^2M_B\tilde{M}\tilde{M}_C\mathscr{G}_1+2a^2M_B\tilde{M}\left[M_f\left((1+2M_k+2M_h)r^*+2(\tilde{M}_k+\tilde{M}_h)\right)+\tilde{M}_f\right] \\
& +2aM_B\tilde{M}Tr(Q)[M_\sigma r^*+\tilde{M}_\sigma]+2mM_B\tilde{M}\Big[\sum_{k=1}^m\beta_kr^*+\sum_{k=1}^m\tilde{\beta}_k\Big]\Big\}\le r^*
\end{aligned}
$$

$$
\begin{aligned}
\text{and }\ \nu^* &= 7\big\{(1+6a^2M_B\tilde{M}\tilde{M}_C\tilde{M}_W)(C_1+C_2+C_3+C_4+C_5)\big\}, \\
\text{where } C_1 &= M_B\tilde{M}\tilde{M}_BM_H,\ C_2=M_B\tilde{M}M_1M_H+M_BM_1, \\
C_3 &= a^2M_B\tilde{M}M_f(1+M_k+M_h),\ C_4=aM_B\tilde{M}Tr(Q)M_\sigma, \\
C_5 &= mM_B\tilde{M}\sum_{k=1}^m\beta_k.
\end{aligned}
$$

Theorem 3. *If the assumptions* $(H3)(ii)-(H7)$ *and* $(H16)-(H20)$ *are satisfied, then the system* (11) *is controllable on* J *provided*

$$7\big\{(1+6a^2M_B\tilde{M}\tilde{M}_C\tilde{M}_W)(C_1+C_2+C_3+C_4+C_5)\big\}<1.$$

Proof: We define the control operator by using the hypothesis $(H2)$

$$
\begin{aligned}
u(t)=\tilde{W}^{-1}\Big[& x_1-B^{-1}\mathscr{S}(a)\left[Bx_0-BH(x)-g(0,x(0))\right]-B^{-1}g(a,x(a)) \\
& -\int_0^a B^{-1}\mathscr{S}(a-s)f\Big(s,x(s),\int_0^s k\big(s,\tau,x(\tau)\big)d\tau,\int_0^a h\big(s,\tau,x(\tau)\big)d\tau\Big)ds \\
& -\int_0^a B^{-1}\mathscr{S}(a-s)\sigma(s,x(s))dw(s)-\sum_{0<t_k<a}B^{-1}\mathscr{S}(a-t_k)I_k(x(t_k^-))\Big](t).
\end{aligned}
$$

Let $\mathscr{Y}_r^*$ be a nonempty closed subset of $\mathscr{P}C(J,\mathscr{L}_2)$ defined by

$$\mathscr{Y}_r^*=\{x : x\in\mathscr{P}C(J,\mathscr{L}_2)|E\|x(t)\|^2\le r^*\}.$$

Consider a mapping $\Psi^* : \mathscr{Y}_r^*\to\mathscr{Y}_r^*$ defined by

$$
\begin{aligned}
(\Psi^*x)(t) \ = \ & B^{-1}\mathscr{S}(t)[Bx_0-BH(x)-g(0,x(0))]+B^{-1}g(t,x(t))+\int_0^t B^{-1}\mathscr{S}(t-s)Cu(s)ds \\
& +\int_0^t B^{-1}\mathscr{S}(t-s)f\Big(s,x(s),\int_0^s k\big(s,\tau,x(\tau)\big)d\tau,\int_0^a h\big(s,\tau,x(\tau)\big)d\tau\Big)ds \\
& +\int_0^t B^{-1}\mathscr{S}(t-s)\sigma(s,x(s))dw(s)+\sum_{0<t_k<t}B^{-1}\mathscr{S}(t-t_k)I_k(x(t_k^-)).
\end{aligned}
$$

All the functions involved in the operator are continuous therefore Ψ^* is continuous. From our assumptions we can evaluate

$$E\|u_x(t)\|^2\le\ 8\tilde{M}_W\Big\{\|x_1\|^2+M_B\tilde{M}\tilde{M}_B\|x_0\|^2+M_B\tilde{M}\tilde{M}_B\tilde{M}_H+2M_B\tilde{M}\left[M_2(\|x_0\|^2+\tilde{M}_H)+M_3\right]$$

$$+M_B(M_2r^*+M_3)+2a^2M_B\tilde{M}\left[M_f\left((1+2M_k+2M_h)r^*+2(\tilde{M}_k+\tilde{M}_h)\right)+\tilde{M}_f\right]$$
$$+2aM_B\tilde{M}Tr(Q)[M_\sigma r^*+\tilde{M}_\sigma]+2mM_B\tilde{M}\Big[\sum_{k=1}^m\beta_k r^*+\sum_{k=1}^m\tilde{\beta}_k\Big]\Big\} := \mathscr{G}_1.$$

$$E\|u_x(t)-u_y(t)\|^2 \le 6\tilde{M}_W\Big\{M_B\tilde{M}\tilde{M}_BM_H+M_B\tilde{M}M_1M_H+M_BM_1+aM_B\tilde{M}Tr(Q)M_\sigma$$
$$+a^2M_B\tilde{M}M_f(1+M_k+M_h)+mM_B\tilde{M}\sum_{k=1}^m\beta_k\Big\}\|x-y\|^2.$$

We first show that the operator Ψ^* maps $\mathscr{Y}_r^*$ into $\mathscr{Y}_r^*$.

$$E\|\Psi^*x(t)\|^2 \le 8\Big\{M_B\tilde{M}\tilde{M}_B(\|x_0\|^2+\tilde{M}_H)+2M_B\tilde{M}\left[M_2(\|x_0\|^2+\tilde{M}_H)+M_3\right]+M_B[M_2r^*+M_3]$$
$$+a^2M_B\tilde{M}\tilde{M}_C\mathscr{G}_1+2a^2M_B\tilde{M}\left[M_f\left((1+2M_k+2M_h)r^*+2(\tilde{M}_k+\tilde{M}_h)\right)+\tilde{M}_f\right]$$
$$+2aM_B\tilde{M}Tr(Q)[M_\sigma r^*+\tilde{M}_\sigma]+2mM_B\tilde{M}\Big[\sum_{k=1}^m\beta_k r^*+\sum_{k=1}^m\tilde{\beta}_k\Big]\Big\}.$$

From $(H20)$ we obtain $E\|(\Psi^*x)(t)\|^2 \le r^*$. Hence Ψ^* maps $\mathscr{Y}_r^*$ into itself. Let $x, y \in \mathscr{Y}_r^*$, then we have

$$E\|\Psi^*x(t)-\Psi^*y(t)\|^2 \le 7\{(1+6a^2M_B\tilde{M}\tilde{M}_C\tilde{M}_W)\big[M_B\tilde{M}\tilde{M}_BM_H+M_B\tilde{M}M_1M_H+M_BM_1$$
$$+a^2M_B\tilde{M}M_f(1+M_k+M_h)+aM_B\tilde{M}Tr(Q)M_\sigma+mM_B\tilde{M}\sum_{k=1}^m\beta_k\big]\}\|x-y\|^2$$
$$\le 7\{(1+6a^2M_B\tilde{M}\tilde{M}_C\tilde{M}_W)(C_1+C_2+C_3+C_4+C_5)\}\|x-y\|^2 \le \nu^*\|x-y\|^2.$$

Since $\nu^* < 1$, the mapping Ψ^* is a contraction and hence there exists a unique fixed point $x \in \mathscr{Y}_r^*$ such that $(\Psi^*x)(t) = x(t)$. This fixed point is then the solution of the system (11) and clearly, $x(a) = (\Psi^*x)(a) = x_1$ which implies that the system (11) is controllable on J.

5. Example

Consider the following partial integrodifferential equation of the form

$$\partial\Big[(z(t,y) - z_{yy}(t,y))-\frac{1}{2}\cos z(t,y)\Big] = \Big(-\frac{\partial^2}{\partial y^2}\Big[z(t,y)+\int_0^t l(t-s)z(s,y)ds\Big]+\mu(t,y)$$
$$+\frac{z(t,y)}{(1+t^2)}+z(t,y)\int_0^t\frac{e^{-z(s,y)}}{(1+t^2)(1+s)}ds+\int_0^1\frac{\sin z(s,y)}{(1+t^2)(1+s)}ds\Big)\partial t$$
$$+\frac{1}{2}e^{-t}(1+t^2)z(t,y)dw(t), \quad t \in J := [0,1],\ y \in [0,1],$$
$$z(t,0) = z(t,1) = 0, \qquad t \in J,$$

$$z(0,y) \quad + \quad \sum_{i=1}^{p} c_i z(t_i,y) = z_0(y), \quad y \in [0,1], \tag{13}$$

where $0 < t_1 < t_2 \cdots < t_p < 1$, p is a positive integer and $z_0(y) \in H$. Take $H = K = U = L^2([0,1])$ and define the operators $A : D(A) \subset H \to H$ and $B : D(B) \subset H \to H$ by

$$Az = -z_{yy}, \quad Bz = z - z_{yy},$$

respectively, where each domain $D(A), D(B)$ is given by

$$D(A) = D(B) = \{z \in H, z, z_y \text{ are absolutely continuous}, z_{yy} \in H \text{ and } z(0) = z(1) = 0\}.$$

Then A and B can be written as

$$\begin{aligned} Az &= \sum_{n=1}^{\infty} n^2 (z,z_n) z_n, \quad z \in D(A), \\ Bz &= \sum_{n=1}^{\infty} (1+n^2)(z,z_n) z_n, \quad z \in D(B), \end{aligned}$$

where $z_n(s) = \sqrt{\frac{2}{\pi}} \sin ns, n = 1, 2, \cdots$ is the orthogonal set of eigenvectors of A. Furthermore, for $z \in H$ we have

$$\begin{aligned} B^{-1}z &= \sum_{n=1}^{\infty} \frac{1}{(1+n^2)} (z,z_n) z_n, \\ AB^{-1}z &= \sum_{n=1}^{\infty} \frac{n^2}{1+n^2} (z,z_n) z_n, \\ T(t)z &= \sum_{n=1}^{\infty} e^{\frac{-n^2}{n^2+1} t} (z,z_n) z_n. \end{aligned}$$

It is easy to see that AB^{-1} generates a strongly continuous semigroup $T(t)$ for $t \geq 0$ on H such that $\|T(t)\| \leq e^{-t}$, $t > 0$. It is well known from [12] that the integrodifferential system (6) has an associated resolvent operator $R(t)$ such that $\|R(t)\| \leq e^{-t}$ for $t > 0$ and the function $F(t) = l(t)$ is continuous and bounded for $t > 0$ which satisfies $(H1)$. Assume that the operator $W : L^2(J,U) \to H$ defined by

$$Wu = \int_0^1 B^{-1} R(1-s) \mu(s,y) ds$$

has an bounded invertible operator W^{-1} which takes values in $L^2(J,U)/KerW$ and satisfies condition $(H2)$ for $y \in [0,1]$.

Put $x(t) = z(t,\cdot)$ and $u(t) = \mu(t,\cdot)$ where $\mu : J \times [0,1] \to [0,1]$ is continuous,

$$g(t,x(t)) = \frac{1}{2} \cos z(t,y), \qquad \sigma(t,x(t)) = \frac{1}{2} e^{-t} (1+t^2) z(t,y),$$

$$
\begin{aligned}
q(t_1, t_2, \cdots, t_p, x(t_1), x(t_2), \cdots, x(t_p)) &= \sum_{i=1}^{p} c_i z(t_i, y), \\
f\left(t, x(t), \int_0^t k(t,s,x(s))ds, \int_0^a h(t,s,x(s))ds\right) &= \frac{z(t,y)}{(1+t^2)} + z(t,y)\int_0^t \frac{e^{-z(s,y)}}{(1+t^2)(1+s)}ds \\
&\quad + \int_0^1 \frac{\sin z(s,y)}{(1+t^2)(1+s)}ds.
\end{aligned}
$$

With this choice of $A, B, f, g, h, k, q, \sigma$, $C = I$, the identity operator and $w(t)$, one dimensional standard wiener process, the equation (13) can be written in the abstract formulation of the system (5). Further we have

$$
\left\| \frac{z(t,y)}{(1+t^2)} + z(t,y)\int_0^t \frac{e^{-z(s,y)}}{(1+t^2)(1+s)}ds + \int_0^1 \frac{\sin z(s,y)}{(1+t^2)(1+s)}ds \right\| \leq \frac{1}{1+t^2}(1+2\log 2)\|z\|.
$$

Further all the other assumptions $(H3)-(H9)$ are obviously satisfied and it is possible to choose c_i's in such a way that the constant $\delta < 1$. Hence, by Theorem 1, the system (13) is controllable on J.

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