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Relative Spaces as a global concept for Topology

Dieter Leseberg

Ernst Reuter Gesellschaft Berlin, Germany

Abstract. In the past several mathematical concepts were introduced and studied for describing structures of a topological kind. Among them we mention here the strong topological universe of preuniform convergence spaces in the sense of Preuss, which enables to simultaneously express "topological" and "uniform" aspects. The introduced concept of set-convergences by Wyler considers the convergence of filters to bounded subsets, and therefore it generalizes the well-known classical point-convergences as well as the supertopologies in the sense of Doîtchinóv. Nearness, defined by Herrlich contains in particular contiguities and proximities by studying the internal relationship of sets which are being near in some special sense. At last we still mention the concept of so-called b-topological spaces in which hull-operators are defined on bounded subsets of a carrier set generalizing topological closures in a natural way. Now, the question raises whether there exists a suitable concept for a common study of all former cited structures? At this point we introduce the basics of so-called relative spaces, shortly RELspaces with its corresponding relative maps, shortly RELmaps between them. Hence it is possible to embed the mentioned categories into REL, the category of RELspaces and RELmaps, respectively. Now, this fact of unification being established we turn us towards the study of the relationship between suitable RELspaces and the corresponding strict symmetric topological extensions. Thus, our new presented connection generalizes those one which were studied by Bentley, Ivanova and Ivanov, Lodato and Smirnov, respectively.

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1. Introduction

Fundamental concepts for describing structures of a *topological* kind can be *essentially* summarized by the following constructs, i.e.

by the UNIFORM CONVERGENCE with the containing concepts of *uniformities* and *point-convergences*, respectively (*see* Preuß, Kent);

by the SET-CONVERGENCE with the containing concepts of *point-convergences* and *neighborhoods* or *supertopologies*, respectively (*see* Tozzi, Wyler, Doîtchinóv);

Email address: `leseberg@zedat.fu-berlin.de`

by the NEARNESS with the containing concepts of contiguities and proximities, respectively (*see* Ivanova + Bentley + Herrlich + Lodato);

but last not least by the HULLNESS with the containing concepts of *closures* and *b-topologies*, respectively (*see* Čech + Kuratowski + Leseberg).

Here, we should note that the above mentioned drafts *differs* from each other and if possible only *coincide* in some *special* cases. But as next we will introduce a *common* concept in which *all* the former mentioned species then can be described as *special* parts.

2. Unification

We call a triple $(X, \mathcal{B}^X, r)$ consisting of a set X, *boundedness* $\mathcal{B}^X$ and a *Relative-operator*, shortly *RELop* $r : \mathcal{B}^X \longrightarrow \underline{P}(REL(X))$ *RELative space*, shortly *RELspace* provided r satisfies the following conditions, i.e.

(relop$_1$) $B \in \mathcal{B}^X \backslash \{\emptyset\}$ and $\mathcal{R}_1 << \mathcal{R} \in r(B)$ imply $\mathcal{R}_1 \in r(B)$;

(relop$_2$) $B \in \mathcal{B}^X$ implies $\{\emptyset\} \notin r(B)$;

(relop$_3$) $\mathcal{R} \in r(\emptyset)$ iff $\mathcal{R} = \emptyset$;

(relop$_4$) $x \in X$ implies $\{\{x\} \times \{x\}\} \in r(\{x\})$.

Here, $\mathcal{B}^X \subset \underline{P}X$ (power set of X) denotes a *collection* of *bounded* subsets of X, known as $\underline{\text{B}}$-set or boundedness on X, respectively, i.e. $\mathcal{B}^X$ has the following properties:

(b$_0$) $\emptyset \in \mathcal{B}^X$;

(b$_1$) $B_1 \subset B \in \mathcal{B}^X$ imply $B_1 \in \mathcal{B}^X$;

(b$_2$) $x \in X$ implies $\{x\} \in \mathcal{B}^X$.

REL(X) denotes the set of all *Relative-systems* $\mathcal{R} \subset \underline{P}(X \times X)$, shortly *RELsystems* (for X), *ordered* by setting: $\mathcal{R}_1 << \mathcal{R}$ iff $\forall R_1 \in \mathcal{R}_1 \exists R \in \mathcal{R}\ R_1 \supset R$. For $B \in \mathcal{B}^X, \mathcal{R} \in r(B)$ then is called B-RELsystem (for r). Moreover, for $\rho \subset \underline{P}X$ $\rho \times \rho$ is defined by $\rho \times \rho := \{F \times F : F \in \rho\}$, where $F \times F := \{(x, z) \in X \times X : x, z \in F\}$.
For RELspaces $(X, \mathcal{B}^X, r)$, $(Y, \mathcal{B}^Y, s)$ a function $f : X \longrightarrow Y$ is called *RELative-map*, shortly *RELmap*, provided f satisfies the following conditions, i.e.

(rmap$_1$) $B \in \mathcal{B}^X$ implies $f[B] \in \mathcal{B}^Y$, which means f is *bounded*;

(rmap$_2$) $B \in \mathcal{B}^X$ and $\mathcal{R} \in r(B)$ imply $f^\times \mathcal{R} \in s(f[B])$, where $f^\times \mathcal{R} := \{(f \times f)[R] : R \in \mathcal{R}\}$ with $(f \times f)[R] := \{(f \times f)(x, z) : (x, z) \in R\} = \{(f(x), f(z)) : (x, z) \in R\}$.

Now, by **REL** we denote the topological construct, whose *objects* are the RELspaces and as *morphisms* the RELmaps between them. Additionally, we point out that each RElspace $(X, \mathcal{B}^X, r)$ has an *underlying hull-operator* "cl_r" defined by $cl_r(\emptyset) := \emptyset$ and $cl_r(A) := \{x \in X : \exists \mathcal{R} \in r(\{x\}) A \times A \in \mathcal{R}\}$, if $A \neq \emptyset$.

Theorem 1. *The* strong topological *universe* ***PUCONV*** *of preuniform convergence spaces and uniformly continuous functions is isomorphic to a* suitable *subcategory of* ***REL***.

Proof. The embedding of **PUCONV** into **REL** can be described as follows: For a given preuniform convergence space (X, J_X) we define the *associated* RELspace $(X, \underline{P}X, r_{J_X})$ by *setting*:

$r_{J_X}(\emptyset) := \{\emptyset\}$ and

$r_{J_X}(B) := \{\mathcal{R} \in REL(X) : \exists \mathcal{U} \in J_X \; \mathcal{R} \subset \sec \mathcal{U}\}$ for each $B \in \underline{P}X \setminus \{\emptyset\}$.

Here, $\sec \mathcal{U} := \{R \subset X \times X : \forall U \in \mathcal{U} \; R \cap U \neq \emptyset\}$.

Theorem 2. *The topological construct* ***SETCONV*** *of set-convergence spaces and b-continuous maps is isomorphic to a* suitable *subcategory of* ***REL***.

Proof. For a set-convergence space $(X, \mathcal{B}^X, q)$ let us consider the RELop $r_q : \mathcal{B}^X \longrightarrow \underline{P}(REL(X))$ defined by setting: $r_q(\emptyset) := \{\emptyset\}$ and $r_q(B) := \{\mathcal{R} \in REL(X) : \exists \mathcal{F} \in FIL(X)((\mathcal{F}, B) \in q$ and $\mathcal{R} \subset \sec \mathcal{F} \otimes \mathcal{F})\}$ for each $B \in \mathcal{B}^X \backslash \{\emptyset\}$, where in *general* for filters $\mathcal{F}_1, \mathcal{F} \in FIL(X)$ $\mathcal{F}_1 \otimes \mathcal{F}$ is defined by $\mathcal{F}_1 \otimes \mathcal{F} := \{R \subset X \times X : \exists F_1 \in \mathcal{F}_1 \exists F \in \mathcal{F} \; R \supset F_1 \times F\}$.

Theorem 3. *The topological construct* ***PNEAR*** *of pre-nearness spaces and n-maps is isomorphic to a* suitable *full subcategory of* ***REL***.

Proof. We construct the embedding of **PNEAR** into **REL** as follows: For a given pre-nearness space (X, ξ) we define the *associated* RELspace $(X, \underline{P}X, r_\xi)$ by setting:

$r_\xi(\emptyset) := \{\emptyset\}$ and

$r_\xi(B) := \{\mathcal{R} \in REL(X) : \exists \mathcal{M} \subset \underline{P}X(\{B\} \cup \mathcal{M} \in \xi$ and $\mathcal{R} << \mathcal{M} \times \mathcal{M})\}$ for each $B \in \underline{P}X \setminus \{\emptyset\}$.

As next, we will consider RELspaces $(X, \mathcal{B}^X, r)$ with *underlying* bornology $\mathcal{B}^X$. Note, that $\underline{P}X$ defines *itself* a bornology, and the set of *finite* sets of X, the set of *compact sets* of X or the set of *total bounded* sets of X defines bornologies on X, too. Now, if given a bornology $\mathcal{B}^X$ and a fixed point $x \in X$, then we can define a hull-operator $h^x : \mathcal{B}^X \longrightarrow \underline{P}X$ by setting: $h^x(\emptyset) := \emptyset$ and $h^x(B) := \{x\} \cup B$ for each $B \in \mathcal{B}^X \backslash \{\emptyset\}$. Then h^x satisfies the axioms for being a so-called *b-topology* t (on $\mathcal{B}^X$), i.e.

(b-top$_1$) $B \in \mathcal{B}^X$ implies $t(B) \in \mathcal{B}^X$;

(b-top$_2$) $B \in \mathcal{B}^X$ implies $B \subset t(B)$;

(b-top$_3$) $t(\emptyset) = \emptyset$;

(b-top$_4$) $B_1 \subset B \in \mathcal{B}^X$ imply $t(B_1) \subset t(B)$;

(b-top$_5$) $B \in \mathcal{B}^X$ implies $t(t(B)) \subset t(B)$;

(b-top$_6$) $B_1, B_2 \in \mathcal{B}^X$ imply $t(B_1 \cup B_2) \subset t(B_1) \cup t(B_2)$.

Then the triple $(X, \mathcal{B}^X, t)$ is called *b-topological space*. Note, that in case if $\mathcal{B}^X$ is *saturated*, which means $X \in \mathcal{B}^X$, hence $\mathcal{B}^X$ *equals* $\underline{P}X$, t is defining a *Kuratowski closure* operator on $\underline{P}X$. *b-continuous* maps between b-topological spaces then can be defined in an *obvious* way.

Theorem 4. *The construct **b-TOP** of b-topological spaces and b-continuous maps is isomorphic to a* suitable *full subcategory of **REL**.*

Proof. For a b-topological space $(X, \mathcal{B}^X, t)$ let us consider the RELspace $(X, \mathcal{B}^X, r_t)$, where r_t is defined by setting:

$r_t(\emptyset) := \{\emptyset\}$ and

$r_t(B) := \{\mathcal{R} \in REL(X) : \exists \rho \subset \mathcal{B}^X \exists x \in B(\mathcal{R} << \rho \times \rho \text{ and } x \in \bigcap\{t(F) : F \in \rho\})\}$ for each $B \in \mathcal{B}^X \setminus \{\emptyset\}$.

3. Extension

Now, an *important* investigated problem in "global topology" is that of considering *topological extensions* of *suitable basic* structures. Here, we should mention the *earlier* studies of Banaschewski, Bentley, Doîtchinóv, Herrlich, Hušek, Ivanova, Ivanov, Lodato, Smirnov and others.

Definition 1. *By adjusting the* fundamental *definition of a topological extension to our* general *concept of "Bounded Topology" we introduce the category **BTEXT**, whose objects are triples* $(e, \mathcal{B}^X, Y)$ *– called* b-topological extensions *– where* $\mathcal{B}^X$ *is* $\underline{B}$*-set,* $X := (X, t_X)$ *and* $Y := (Y, t_Y)$ *are topological spaces, given by closure operators, and* $e : X \longrightarrow Y$ *is function such that the following properties are satisfied, i.e.*

(btex$_1$) $\{t_Y(e[A]) : A \subset X\}$ *forms a* base *for the* closed *subsets of* Y*;*

(btex$_2$) $t_Y(e[X]) = Y$*, which means that the image of* X *under* e *is* dense *in* Y*;*

(btex$_3$) $x \in X$ *and* $y \in t_Y(\{e(x)\})$ *imply* $e(x) \in t_Y(\{y\})$, point-symmetry *of* t_Y *with respect to e;*

(btex$_4$) $A \in \underline{P}X$ *implies* $t_X(A) = e^{-1}[t_Y(e[A])]$*, where* e^{-1} *denotes the* inverse *image of* e.

Morphisms *in* ***BTEXT*** *have the form* $(f,g) : (e, \mathcal{B}^X, Y) \longrightarrow (e', \mathcal{B}^{X'}, Y')$, *where* $f : X \longrightarrow X'$ *and* $g : Y \longrightarrow Y'$ *are continuous maps such that* f *is bounded, and the following diagram commutes, i.e.*

$$\begin{tikzcd} X \arrow[r, "e"] \arrow[d, "f"'] & Y \arrow[d, "g"] \\ X' \arrow[r, "e'"] & Y' \end{tikzcd} .$$

If $(f,g) : (e, \mathcal{B}^X, Y) \longrightarrow (e', \mathcal{B}^{X'}, Y')$ *and* $(f', g') : (e', \mathcal{B}^{X'}, Y') \longrightarrow (e'', \mathcal{B}^{X''}, Y'')$ *are* ***BTEXT****-morphisms, then they can be composed according to the rule:*

$$(f', g') \diamond (f, g) := (f' \circ f, g' \circ g) : (e, \mathcal{B}^X, Y) \longrightarrow (e'', \mathcal{B}^{X''}, Y''),$$

where "$\circ$" denotes the composition *of maps. Further, we note that in the case if* $\mathcal{B}^X$ *is saturated the above definition* coincide *with the* usual *one of a* strict *topological extension in "classical topology". Observe too, that the axiom (btex$_4$) in this definition is* automaticially *satisfied if* $e : X \longrightarrow Y$ *is a* topological embedding.

Lemma 1. *Let* $(e, \mathcal{B}^X, Y)$ *be a b-topological extension, then we can* immediately *associate the RELspace* $(X, \mathcal{B}^X, r_e)$, *where* r_e *is defined by setting:*

$r_e(\emptyset) := \{\emptyset\}$ *and*

$r_e(B) := \{\mathcal{R} \in REL(X) : \exists \mathcal{M} \subset \underline{P}X(\mathcal{R} << \mathcal{M} \times \mathcal{M}$ *and* $\bigcap\{t_Y(e[A]) : A \in \mathcal{M} \cup \{B\}\}\} \neq \emptyset)\}$ *for each* $B \in \mathcal{B}^X \backslash \{\emptyset\}$.

Moreover, we are getting the equality $cl_{r_e}(A) = t_X(A)$ *for each* $A \in \underline{P}X$.

Corollary 1. *The RELspace* $(X, \mathcal{B}^X, r_e))$ *satisfies in* addition *the conditions for being a* nearformic CLANRELspace, *which* concretely *means that* r_e *possesses the following properties, i.e.*

(relop$_5$) $\emptyset \neq B_1 \subset B \in \mathcal{B}^X$ *imply* $r(B_1) \subset r(B)$;

(relop$_6$) $\mathcal{R} \in r(B), B \in \mathcal{B}^X \backslash \{\emptyset\}$ *imply the existence of* $\rho \subset \underline{P}X$ *with* $\mathcal{R} << \rho \times \rho \in r(B)$;

(relop$_7$) $\rho \subset \underline{P}X$ *and* $cl_r\rho \times cl_r\rho \in r(B), B \in \mathcal{B}^X \backslash \{\emptyset\}$ *imply* $\rho \times \rho \in r(B)$, *where* $cl_r\rho := \{cl_r(F) : F \in \rho\}$;

(relop$_8$) $\rho_1, \rho_2 \subset \underline{P}X$ *and* $(\rho_1 \vee \rho_2) \times (\rho_1 \vee \rho_2) \in r(B), B \in \mathcal{B}^X \backslash \{\emptyset\}$ *imply* $\rho_1 \times \rho_1 \in r(B)$ *or* $\rho_2 \times \rho_2 \in r(B)$, *where* $\rho_1 \vee \rho_2 := \{F_1 \cup F_2 : F_1 \in \rho_1, F_2 \in \rho_2\}$;

(cla) $\mathcal{R} \in r(B), B \in \mathcal{B}^X \backslash \{\emptyset\}$ *imply the existence of a B-RELclan* $\mathcal{C}$ *in* r *with* $\mathcal{R} << \mathcal{C} \times \mathcal{C}$;

(n) $\mathcal{M} \subset \underline{P}X$ *and* $\mathcal{M} \times \mathcal{M} \in r(B), B \in \mathcal{B}^X \backslash \{\emptyset\}$ *imply* $(\{B\} \cup \mathcal{M}) \times (\{B\} \cup \mathcal{M}) \in \bigcap\{r(D) : D \in \{B\} \cup (\mathcal{M} \bigcap \mathcal{B}^X)\}$.

Remark 1. *Here, for $B \in \mathcal{B}^X \backslash \{\emptyset\}$, $\mathcal{C} \subset \underline{P}X$ is called* B-RELclan *(in r) provided the following conditions are satisfied, i.e.*

(rcla$_1$) $\emptyset \notin \mathcal{C}$;

(rcla$_2$) $D_1 \cup D_2 \in \mathcal{C}$ *iff* $D_1 \in \mathcal{C}$ *or* $D_2 \in \mathcal{C}$;

(rcla$_3$) $B \in \mathcal{C}$;

(rcla$_4$) $\mathcal{C} \times \mathcal{C} \in r(B)$;

(rcla$_5$) $cl_r(D) \in \mathcal{C}, D \in \underline{P}X \setminus \{\emptyset\}$ *imply* $D \in \mathcal{C}$.

We point out that for each $B \in \mathcal{B}^X$ with $x \in B$ $x_r := \{D \in \underline{P}X : x \in cl_r(D)\}$ is B-RELclan in r.
Further we denote by ***N-CLANREL*** *the full subcategory of* ***REL****, whose objects are the nearformic CLANRELspaces.*

Theorem 5. *Let F:* ***BTEXT*** $\longrightarrow$ ***N-CLANREL*** *be defined by:*

(1) For a ***BTEXT****-object* $(e, \mathcal{B}^X, Y)$ *we put* $F(e, \mathcal{B}^X, Y) := (X, \mathcal{B}^X, r_e)$;

(2) for a ***BTEXT****-morphism* $(f, g) : (e, \mathcal{B}^X, Y) \longrightarrow (e', \mathcal{B}^{X'}, Y')$ *we put* $F(f, g) := f$.

*Then F:***BTEXT** $\longrightarrow$ ***N-CLANREL*** *is a functor.*
Now, this former being established we are going to introduce a related *functor in the* opposite *direction.*

Theorem 6. *We obtain a functor G:****N-CLANREL*** $\longrightarrow$ ***BTEXT*** *by setting:*

(i) $G(X, \mathcal{B}^X, r) := (e_X, \mathcal{B}^X, X^C)$ *for any nearformic CLANRELspace* $(X, \mathcal{B}^X, r)$ *with* $X := (X, cl_r)$ *and* $X^C := (X^C, t_{X^C})$, *where* $X^C := \{\mathcal{C} \subset \underline{P}X : \mathcal{C}$ *is B-RELclan in r for some* $B \in \mathcal{B}^X \backslash \{\emptyset\}\}$, *and for each* $A^C \subset X^C$ *we put:*

$$t_{X^C}(A^C) := \{\mathcal{C} \in X^C : \triangle A^C \subset \mathcal{C}\} \text{ with } \triangle A^C := \{F \in \underline{P}X : \forall \mathcal{C} \in A^C \ F \in \mathcal{C}\}.$$

(By convention $\triangle A^C = \underline{P}X$ *if* $A^C = \emptyset$*).* $e_X : X \longrightarrow X^C$ *denotes that function which assigns the* $\{x\}$*-RELclan* x_r *to each* $x \in X$.

(ii) $G(f) := (f, f^C)$ *for any RELmap* $f : (X, \mathcal{B}^X, r) \longrightarrow (Y, \mathcal{B}^Y, s)$, *where* $f^C : X^C \longrightarrow Y^C$ *is the function defined by setting:*

$$f^C(\mathcal{C}) := \{\mathcal{D} \in \underline{P}Y : f^{-1}[cl_s(D)] \in \mathcal{C}\} \text{ for each } \mathcal{C} \in X^C.$$

Theorem 7. *Let* F : ***BTEXT*** $\longrightarrow$ ***N-CLANREL*** *and* G : ***N-CLANREL*** $\longrightarrow$ ***BTEXT*** *be the above defined functors.*
For each object $(X, \mathcal{B}^X, r)$ *of* ***N-CLANREL*** *let* $n_{(\mathcal{B}^X, r)}$ *denote the identity map* $id_X : F(G(X, \mathcal{B}^X, r)) \longrightarrow (X, \mathcal{B}^X, r)$. *Then* $n : F \circ G \longrightarrow \underline{1}_{\textbf{N-CLANREL}}$ *is a* natural equivalence *from* $F \circ G$ *to the identity functor* $\underline{1}_{\textbf{N-CLANREL}}$, *i.e.*

$$id_X : F(G(X, \mathcal{B}^X, r)) \longrightarrow (X, \mathcal{B}^X, r)$$

is RELmap in both *directions for each object* $(X, \mathcal{B}^X, r)$*, and the following diagram* commutes *for each RELmap* $f : (X, \mathcal{B}^X, r) \longrightarrow (Y, \mathcal{B}^Y, s)$*, i.e.*

$$\begin{tikzcd} F(G(X,\mathcal{B}^X,r)) \arrow[r, "id_X"] \arrow[d, "F(G(f))"'] & (X,\mathcal{B}^X,r) \arrow[d, "f"] \\ F(G(Y,\mathcal{B}^Y,s)) \arrow[r, "id_Y"] & (Y,\mathcal{B}^Y,s) \end{tikzcd}$$

Corollary 2. *For a b-topological* T_1-extension $(e, \mathcal{B}^X, Y)$*, where in* addition (Y, t_Y) *is* T_1-*space, then the associated RELspace* $(X, \mathcal{B}^X, r_e)$ *is* separated *by satisfying the following condition, i.e.*

(sep) $x, z \in X$ *and* $\{\{z\} \times \{z\}\} \in r(\{x\})$ *imply* $x = z$.

Corollary 3. *For a separated nearformic CLANRELspace* $(X, \mathcal{B}^X, r)$ *the function* $e_X : X \longrightarrow X^C$ *is* injective.

Remark 2. *As a short* résumé *of Theorem 7 we now can state that each nearformic CLANRELspace* $(X, \mathcal{B}^X, r)$ *can be* densely embedded *into a topological space* Y *so that the set of the* closures *of* all *sets in* X *forms a* base *for the* closed *subsets of* Y*. In addition we note that the closures of the members of suitable B-RELsystems* meet *the closure of* B *in* Y*. Thus, Doîtchinóv's "extension theorem" can be considered as a* special *part of this theorem. Moreover, in the* saturated *case the theorem* essentially coincide *with that of Bentley, presented by him in the past using bunch-determined nearness spaces. Hence, Lodatos' theorem and that of Ivanova also can be dealt with!*

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