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Pricing Policy with Time and Price Dependent Demand for Deteriorating Items

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Abstract. The market of a product is stochastic in nature, especially in terms of demand and price. If demand is high in short span of time, the price also rises proportionately, but demand highly depends on consumer's need. In diminishing market, demand of a product decreases and due to this, product may disappear altogether from the market. One can opt out and reduce the selling price and generate excess demand to earn more and to establish the product in market. In competitive environment, the strategy is also applicable in entering into competition with others. The objectives of present paper are to develop a dynamic pricing policy to solve such types of problems in a diminishing market. The problem is solved by coming to terms with 'Kuhn Tucker imperatives and modalities', in this regard. A simulation study is appended to measure the effect of various parameters on optimal policy. The analysis reveals that for every business setup, there will be an optimal number of price settings for dynamic pricing policy that outperforms the static pricing policy.

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Key Words and Phrases: Inventory, Deterioration, Dynamic Pricing, Time dependent price sensitive demand, Optimal number of price settings

1. Introduction

In a competitive environment retailer and item producing company both identifies the importance of pricing policy so as to improve the revenue and earn more profit. A perfect pricing and marketing policy may boost the company's bottom-line. After a time duration some products like fashion apparels, cosmetic, winter wear etc. are out dated or completely perished. To solve such problem company management needs to design a pricing policy in such a way that the entire stock be sold out before entering into the next cycle. For this, the company may go with a special sale, price discount, stock display or continuous price decay. Inventory cost plays a vital role in inventory management. Expenditure sources like

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ordering cost, safety, lead time and numbers of lots are the integral parts of decision making. An integrated inventory model focusing on these issues has been discussed by [5]. In a contribution, [9] introduced the concept of sale promotion (at the festival) for the clearance of stock and compared two models having without special sale and is with a special sale. They found that the model with special sale outperforms to earlier. The back order and partial lost sale is investigated by [6] with impact of lead time on optimal policy and safety. Some related contribution we refer to [12], [16] and [10]. By dividing the demand rate into segments [13] introduced three component demand rate for newly launched deteriorating item. [8] applied the stock dependent demand theory on for deteriorating items whereas [11] examined same for simple inventory system. [2] had shown that the inventory levels after ordering and price-charge are strategic substitutes. They analyzed simultaneous price and inventory in an incapacitated system by using stochastic demand for single items. The aspect on inflation and delay in payment to vendors has been attempted by many authors. [15] designed the EOQ model for deteriorating item under assuming that demand depends on price and stock. A similar approach followed by [3] on deterministic economic order quantity (EOQ) inventory model by taking into account the inflation and the time value of money for deteriorating items with price and stock-dependent selling rate. some useful contribution due to [4, 8, 14].

Not only price reduction but also price hike specially on fuel, excise duty, transportation increases rate of inflation and many related factors affecting manufacturing, marketing and servicing cost. [3] used a linear demand function with price sensitiveness and allowed retailer to use a continuous increasing price strategy in an inventory cycle. He derived the retailer optimal profit ignoring all inventory cost. His findings are restricted for growing market neither stable nor declining market. A research overview presented by [1] is based on the present problem and for future planning as it jointly determines the dynamic pricing and order level both. [15] presented an economic production quantity model for deteriorating items when the demand rate depends not only on-display stock, but also on the selling price per unit of the item. Due to economic policy, political scenario and agriculture productivity both get affected. [7] dealt with such type of situation and proposed models with uncertain inflation for deteriorating items. [18] showed analytically that solution of vendor managed problem to outperform to the traditional solution of the inventory problem. [17] discussed an inventory policy for products with price and time dependent demand. He obtained the order size and optimal prices both when the decision maker has an opportunity to adjust price before the end of sale season by Kuhn-Tucker's necessary conditions and derived an optimal solution. A large proportion of customers are influenced by advertisements may be through electronic media, newspapers, internet or companion. Rebate in price through advertisements affects sales in supermarkets. Mostly in declining market it happens that reduces constantly and managers put their effort to uplift the sale through media and pricing policy. This conflict motivations for the dynamic behavior based study of the inventory system.

2. Notations and Assumptions

The proposed model has been developed under assumptions that shortages are not allowed and replenishment rate is infinite. Notations bearing the concepts utilized in the discussion are given as under:

L Prescribed time horizon.

N Number of change in selling price.

N_m Maximum number permissible changes in price.

T Time interval for any two price change where $T = L/n$.

s_i Total sale quantity from beginning to end of ith change in prices.

q Quantity required for sale over time horizon L.

h Holding cost unit per unit time is constant.

C_3 Set-up cost.

c Unit purchasing cost.

c_0 Cost arising to change the price once.

b Parameter associative to non increasing (decreasing) trend in demand.

a Initial demand at $t = 0$.

p_j Selling price of product in interval $[(j-1)T, jT]$ where $j = 1, 2, \ldots, n$.

θ Rate of deterioration in system.

β Parameter associate to contribution of price in demand.

$D_i(n, p_n)$ Amount of deteriorated units in time interval $[(i-1)T, iT]$.

$D(n, p_n)$ Amount of total deteriorated units in system over time horizon L.

$H_i(n, p_n)$ Inventory carrying cost over time horizon $(i-1)T$ to iT.

$H_n(n, p_n)$ Total sales revenue over time horizon L.

$R_n(n, p_n)$ Total sales revenue over time horizon L.

$F_n(n, p_n)$ Net profit over time horizon L.

3. Formulation of Proposed Model

Assumes that a product is purchased at rate c per unit for time horizon L. Management follows a strategy to change the selling price n times, therefore time L is divided into n equal parts such that $T = L/n$ and the intervals for price settings are $[0, T]$, $[T, 2T], \ldots,$ $[(n-1)T, nT]$. Management has to decide as to how many times (n) price may change to earn maximum profit and also to determine respective optimal prices (p_i) and ordered quantity (q). If optimal prices for these intervals are $p_1, p_2, \ldots, p_n$, and c_0 be the cost associated to change the selling price once, so total cost for change in selling price is nc_0. If is sold units of product in period $[0, iT]$, where $i = 1, 2, 3, \ldots, n$. In order to reduce the selling price p_j and generate the access demand suppose that the demand of product is:

$$d_t(p_j) = (a - bt - \beta p_j) \tag{1}$$

where $(j-1)T < t \leq jT$, $a - bt - \beta p_j > 0$, $a > 0$, $b > 0$, $\beta > 0$, β, a are fix and known for a given business setup. Suppose sale amount of product is s_i over time interval $[0, iT]$ then

$$s_j = \sum_{i=1}^{i=n} \int_{(j-1)T}^{jT} d_t(p_i)dt - \beta T \sum_{i=1}^{i=n} p_i^2 - \frac{1}{2} i^2 bT^2 \tag{2}$$

The total sold amount in time horizon $L = nT$ is

$$S_n = anT - \beta \sum_{j=1}^{i=n} p_j - \frac{1}{2} n^2 bT^2 \tag{3}$$

The objective is to find out optimal number of change in selling price (n) and respective selling prices p_j, along with optimum profit $F(n, p^n)$.

Suppose $I_i(t, p^n)$ is on hand inventory at time t, and θ is rate of deterioration in interval $[(i-1)T, iT]$. Then rate of decay in inventory is sum of the deteriorated units and demand rate of product per unit time (i.e. $\theta I_i(t, p^n) + d_(i-1)T + t)$). However, the rate of decay in inventory is $\frac{d}{dt} I_i(t, p^n)$ with negative sign. Thus the differential equation would be in following form:

$$\frac{d}{dt} I_i(t, p^n) = -d_{(i-1)T+t} = -(a + bT - biT - bt - \beta p_i) \tag{4}$$

with boundary condition $I_i(0, p^n) = q - s_{i-1}$ and $d_{(i-1)T+t} > 0$

$$I_i(t, p^n) = (q - s_(i-1))(1 - \theta t) - (a_i - biT - \beta p_i)t$$

$$\frac{t^2}{2}(b + a_i\theta - b_i T\theta - \beta\theta p_i) - \frac{1}{6} b\theta t^3 \tag{5}$$

where $a_i = a + bT$. Deteriorated units in interval $(i-1)T, iT$ are $D_i(n, p^n)$

$$D_i(n, p^n) = I_i(T, p^n) - I_i(T, p^n)$$

Total deteriorated units in system over time horizon L is

$$D(n,p^n) = \sum_{i=1}^{n} D_i(n,p^n) \tag{6}$$

The holding cost will be

$$H(n,p^n) = h \sum_{i=1}^{n} \int_0^T I_i(n,p^n)dt \tag{7}$$

Total sale revenue $R(n,p^n)$ over time L is

$$\begin{aligned} R(n,p^n) &= \sum_{i=1}^{n} p_i \int_{(j-1)T}^{jT} d_t(p_i)dt \\ &= \left[aT \sum_{i=1}^{n} p_i - \beta T \sum_{i=1}^{n} {p_i}^2 - \frac{1}{2} bT^2 \sum_{i=1}^{n} (2i-1)p_i \right] \end{aligned} \tag{8}$$

Due to occurrence of deterioration the amount of required stock in system is

$$q_1 = q + D(n,p^n)$$

Net profit $F(n,p^n)$ in this business schedule is

$$F(n,p^n) = R(n,p^n) - H(n,p^n) - q_1 c - nc_0 - C_3 \tag{9}$$

Now we develop the objective function $L(n,p^n)$ as per Kuhn Tucker condition under the condition that

$$p_i < \frac{a - biT}{\beta} \tag{10}$$

$$L(n,p^n) = F(n,p^n) - \sum_{i=1}^{n} \left[p_i - \frac{a - biT}{\beta} + {Z_i}^2 \right] \tag{11}$$

where $A_i = {Z_i}^2$, $n < N_{Max}$

Theorem 1. *For fix n, and $A_i > 0$, optimal price in interval $[(i-1)T, iT]$ is*

$$p_{i_1} = \frac{a}{2\beta} - \frac{b(2i-1)T}{a\beta} + \frac{c}{2}\left(1 - \frac{\theta T}{2}\right) + \frac{hT}{2}\left(i - \frac{1}{2} - \frac{i\theta T}{2} + \frac{\theta T}{6})\right) - \frac{\lambda_i}{2\beta T} \tag{12}$$

If $A_i \le 0$ then

$$p_{i2} = \frac{a - biT}{\beta},$$

where $i = 1, 2, 3, \dots, n*$.

Proof. From equation (11)

$$\frac{\partial}{\partial p^n}L(n,p^n) = aT - 2\beta T p_i - \frac{bT^2}{2}(2i-1) + 2\beta T$$

$$n\left[-\beta T i\left(T^2 - \frac{aT^2}{2}\right) + \frac{\beta T^2}{2} - \frac{\theta\beta T^2}{6}\right] - \lambda_i \tag{13}$$

$$\frac{\partial}{\partial \lambda_i}L(n,p^n) = -\left(p_i - \frac{a-biT}{\beta} + {Z_i}^2\right) = 0, or A_i = \frac{a-biT}{\beta} - p_{i_1}$$

$$\frac{\partial}{\partial Z_i}L(n,p) = -2\lambda_i Z_i = 0 \tag{14}$$

Equations (13), optimize the price, For $A_i = {Z_i}^2 > 0$, we have

$$p_{i_1} = \frac{a}{2\beta} - \frac{b(2i-1)T}{a\beta} + \frac{c}{2}\left(1 - \frac{\theta T}{2}\right) + \frac{hT}{2}\left(i - \frac{1}{2} - \frac{i\theta T}{2} + \frac{\theta T}{6})\right) - \frac{\lambda_i}{2\beta T}.$$

For $A_i = {Z_i}^2 \le 0$, then equation (14) leads to

$$p_{i2} = \frac{a-biT}{\beta} \tag{15}$$

□

Lemma 1. *$F(n,p^n)$ is concave for given n and $i = 1,2,\ldots,n$*

Proof.

$$\frac{\partial^2}{\partial p^2}F(n,p^n) = -2n\beta T \le 0$$

$$\frac{\partial^2}{\partial p^2}F(n,p^n) = -2n\beta T \le 0, \text{ for } i \ne j$$

Then k^{th} principal minor determinates of Hessian matrix (according to Kuhn-Tucker necessary condition) are of sign $(-1)^k$ for $k = 1,2,\ldots,n$. So $F(n,p^n)$ attains global maxima and H is negative definite therefore $F(n,p^n)$ is concave. □

Lemma 2. *$R(n,p^n)$ is concave for given n.*

Proof. from equation (8)

$$\frac{\partial}{\partial {p_i}^2}F(n,p^n) \tag{16}$$

$$\frac{\partial}{\partial {p_i}^2}F(n,p^n) \tag{17}$$

Then k^{th} principal minor determinates of Hessian matrix (according to Kuhn-Tucker necessary condition) are of sign $(-1)^k$ for $k = 1,2,\ldots,n$. So $R(n,p^n)$ exist global maxima and H is negative definite therefore $R(n,p^n)$ is concave. If $R(n,p^n)$ is concave then this shows that revenue $R(n,p^n)$ exists global maxima. That is the solution of proposed problem exists. □

Theorem 2. *In the proposed model price is continuously in decreasing order.*

Proof. For fix n and for each i, there are two possible cases.

Case I If $A_i = {Z_i}^2 > 0$

$$p_{i_1} = \frac{a}{2\beta} - \frac{b(2i-1)T}{a\beta} + \frac{c}{2}\left(1 - \frac{\theta T}{2}\right) + \frac{hT}{2}\left(i - \frac{1}{2} - \frac{i\theta T}{2} + \frac{\theta T}{6})\right) - \frac{\lambda_i}{2\beta T} \quad (18)$$

$$\frac{\partial}{\partial i} p_i = p_i - p_{i-1} = -\frac{bT}{2\beta} - \left(\frac{hT}{2}\frac{\theta T}{2} - 1\right), i > 1 \quad (19)$$

As per laid down $b > 0$, $h > 0$, $\beta > 0$ and $T > 0$.

Case II If $A_i = {Z_i}^2 \leq 0$ from equation (13) $\frac{\partial p_i}{\partial i} = p_i - p_{i-1} = \frac{bT}{\beta}$ which followed the result.

□

Theorem 3. *Fix for n and for each i, $R(n, p^n)$ is monotonic increasing.*

Proof. Using (7)

$$R\left(n, p^n\right) - R\left(n-1, p^{n-1}\right) = aT\beta p_i - \beta T {p_i}^2 - \frac{bT^2}{2}(2i-1)p_i$$

$$i > 1, Tp_i\left(a - \beta p_i - biT + \frac{bT}{2}\right) > 0$$

by using (9), $a - biT - \beta p_i > 0$ Hence $R\left(n, p^n\right)$ is monotonic increasing for fix n, and for every $i = 1, 2, \ldots, n$. The result shows that the selling price is decreasing continuously but even then revenue $R\left(n, p^i\right)$. □

Corollary 1. *$R\left(i, p^i\right)$ is maximal it $i = n$*

Theorem 4. *For fix n and for each $i, F\left(n, p^n\right)$ is monotonic increasing.*

Corollary 2. *For fix n and for each i, $F\left(n, p^n\right)$ is maximum at $i = n$.*

Theorem 5. *For fix n and for each i, s_i is monotonic increasing.*

Proof. Since $s_i = aiT - \beta T \sum_{i=1}^{n} p_j - \frac{bT}{2}$, i.e.

$$s_i - s_{i-1} = a\left(-b(i+1)T - \beta p_i + \frac{3}{2}bT\right), i > 1 \quad (20)$$

As per laid down, demand is $d(t, p_j) = a - bt - \beta p_j > 0$, for $t \in [0, nT], T = L/n$. For $t = (i+1)T, a - b(i+1)T - \beta p_i > 0$

$$s_i - s_{i-1} > 0, i > 1 \quad (21)$$

□

Corollary 3. *For fix n and for each i, s_i is maximum at $i = n$.*

Theorem 6. *For fix n and for each $i, D_i(n, p^n)$ is monotonic decreasing at $i = n$.*

Proof. As per laid down

$$D_i\left(n, p^n\right) = [I_i\left(T, p^n\right)]_{\theta=0} - [I_{i-1}\left(T, p^n\right)]_{\theta=0}$$

$$D_i(n, p^n) = \theta T(q - s_{i-1}) + \frac{\theta T^2}{2}\left(\frac{bT}{3} - biT\beta iT - \beta p_i - aT\right)$$

i.e.

$$D_i(n, p^n) - D_{i-1}(n, p^n) = -\theta\left(s_{i-1} - s_{i-2}\right) - \frac{\theta T^2}{2}\left(a + bT + \beta(p_{i-1} - p_i)\right)$$

As per Theorem 5, $s_{i-1} - s_i > 0$ and by Theorem 2,

$$p_{i-1} - p_i > 0, i > 1$$

$$a + bT + \beta(p_{i+1} - p_i) > 0$$

Hence

$$D_i(n, p^n)\left(D_{i-1}(n, p^n)\right) < 0 \tag{22}$$

□

4. Solution Procedure

i) Calculate A_i

- If $A_i > 0$ then calculate p_{i1} from (11) (Theorem 1),
- If $A_i \le 0$ calculate p_{i2} from (12) (Theorem 1),

ii) Calculate respective $R(n, p^i)$ where $i = 1, 2, 3, \dots, n$

- According to Corollary 1 of theorem 3, $R(n, p^i)$ is maximal at $i = n$.

iii) Calculate $R(n, p^n)$ for each n.

iv) Calculate respective $F(n, p^n)$

- According to Corollary 2 of theorem 4, $F(n, p^i)$ is maximal at $i = n$.

v) Calculate $F(n, p^n)$ for each n.

vi) Repeat above for $n = 1, 2, 3, \dots, N_{Max}$

vii) Examine m for which $F(j, p^j) \le F(m, p^m) \ge F(k, p^k)$, where $j = 1, 2, \dots, (m-1)$ and $k = m, m+1, m+2, \dots, n$. Declare m for which $F(m, p^m)$ is maximal and then optimal price setting will be m. Also declare the corresponding optimal selling prices $p_1, p_2, \dots, p_m$.

5. An Example

Consider the demand function $d_t(p_j) = \left(1000 - 8t - 1.5p_j\right)$ and other parameters are $c_0 = 200$ per price change, $C_3 = 200$ per order, $\theta = 0.01$ unit per unit time, $c = 110$ per unit, $L = 100$ days and $N_{Max} = 12$. We apply the above detailed solution procedure, for $n = 1, 2, \ldots, 12$, and compute the corresponding cost, revenue and profit. Using above one can find the optimal profit from Table 1.

Table 1: Optimal Policy (* indicates the optimal strategy)

n	s_n	$R(np^n)$	Purch. Cost	Total Cost	$F(n,p^n)$	Opti. Pri. for $T = 10$
1	21753	5546525	2392775	2393004	3153320	$p_1 = 375.00$ for $[0,10]$
2	21749	6212779	2392363	2393220	381935	$p_2 = 348.34$ for $(10,20]$
3	21748	6336129	2392256	2393433	3942496	$p_3 = 321.68$ for $(20,30]$
4	21747	6379291	2392208	2393638	3985453	$p_4 = 295.02$ for $(30,40]$
5	21747	6399264	2392181	2393840	4005223	$p_5 = 268.35$ for $(40,50]$
6	21747	6410111	2392164	2394041	4015869	$p_6 = 241.69$ for $(50,60]$
7	21747	6416650	2392152	2394242	4015869	$p_7 = 215.03$ for $(60,70]$
8	21747	6420893	2392137	2394426	4026250	$p_8 = 188.37$ for $(70,80]$
9	21747	6423802	2392137	2394642	4028958	$p_9 = 161.70$ for $(80,90]$
10*	21747*	6425881*	2392132*	2394843*	4030838*	$p_{10} = 135.04$ for $(90,100]$
11	23572	5410769	2592874	2595781	2814787	
12	25345	4470769	2787933	2791038	1679530	

On observing Table 1, an optimal price setting is at $n = 10$ (optimal profit highest at $n = 10$) and for these, corresponding optimal prices are $p_1, p_2, \ldots, p_{10}$. in time intervals $[0,10]$, $(10,20]$, $(20,30]$, $(30,40]$, $(40,50]$, $(50,60]$, $(70,80]$, $(80,90]$ and $(90,100]$.

From Table 1, optimal profit is $F(n,p^n) = 4030828$ at $n = 10$ which is 27% higher than the static pricing policy at $n = 1$ ($F(n,p^n) = 3153320$), which reveals that dynamic pricing policy outperforms the static pricing policy. If we apply a linear function of price [9] then optimal profit will be $F(n,p^n) = 3153320$, which is less than our optimal profit.

Revenue increases till $n = 10$ and thereafter decreases and lowest at $n = 1$ (see Table 1). Noticeable result is that total inventory cost is minimum (2393004) at $n = 1$, than 2394843 at $n = 10$, but optimal profit is high at $n = 10$ than $n = 1$. This reveals the minimum incurred cost does not guarantee for more revenue. However, higher market capitalization indicates for higher level of profit.

6. Conclusion

When demand declines in a market, the implementation of a dynamic optimal pricing policy not only helps stabilize the product in the market but also provides the system with the capability to compete with other products. The numerical example and simulation study both reveal that the proposed dynamic pricing policy outperforms the static pricing policy. Our results also show that revenue continues to increase although the selling price decreases. An increase in purchase cost does not affect the optimal number of price settings, but it does reduce the optimal profit; the same is followed due to parameters b and β. The total inventory cost is lower with the static pricing policy than with the dynamic pricing policy. Lower cost does not guarantee to earn higher profits, but higher volume and large market capitalization do lead to higher revenues and profits. Theoretical and analytical evidence indicates the existence of an optimal number of changes in the value of selling prices for achieving an optimal profit in any business setup. Inventory managers are advised to keep the β parameter high so as to generate excess demand and, in so doing, more revenue. For further interest one, can also relax parameter binding and develop dynamic replenishment policies and dynamic pricing policies as ways in which to adapt to the growing market.

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References

[1] W. Elmaghraby and P. Keskinocak. *Dynamic pricing in the presence of inventory considerations: research overview, current practices and future directions*, Management Science 49, 1287-1309, 2003.

[2] A. Federgruen and A. Heching. *Combined pricing and inventory control under uncertainty*, Operations Research 47, 454-475, 1999.

[3] P. Joglekar. *Optimal price and order quantity strategies for the reseller of a product with price sensitive demand*, Proceedings of the Academy of Information and Management Sciences 7(1), 13-19, 2003.

[4] U.K. Khedlekar and D. Shukla. *Dynamic inventory model with logarithmic demand*, Opsearch 50(1), 1-13, 2013.

[5] M. C. Lo. *Decision support system for the integrated inventory model with general distribution demand*, Information Technology Journal 6(7), 1069-1074, 2007.

[6] M. C. Lo, J. C. H. Pan, K. C. Lin, and J. W. Hsu. *Impact of lead time and safety factor in mixed inventory models with backorder discount*, Journal of Applied Science 8(3), 528-533, 2008.

[7] A. Mirzazadeh. *Effects of uncertain inflationary conditions on an inventory Model for deteriorating items with shortages*, Journal of Applied Science 10(22), 2805-2813, 2010.

[8] G. Padmanabhan and P. Vrat. *EOQ models for perishable items under stock dependent selling rate*, European Journal of Operational Research 86, 281-292, 1995.

[9] T. Roy and K. S. Chaudhuri. *An inventory model for a deteriorating item with price-dependent demand and special sale*, International Journal of Operations Research 2(2), 173 -187, 2007.

[10] H. Sabahno. *Optimal policy for a deteriorating inventory model with finite replenishment rate and with price dependent demand rate and length dependent price*, Proceeding of World Academic Science England Technology 44, 219-223, 2008.

[11] S. M. Seeletse. *Mathematical management of simple inventory system*, Journal of Applied Science 1(1), 260-262, 2001.

[12] N. H. Shah and P. Pandey. *Deteriorating inventory model when demand depends on advertisement and stock display*, International Journal of Operations Research 6(2), 33-44, 2009.

[13] D. Shukla and U. K. Khedlekar. *An order level inventory model with three-Component demand rate (TCDR) for newly launched deteriorating item*, International Journal of Operations Research 7(2), 61-70, 2010.

[14] D. Shukla, U. K. Khedlekar, R. P. S. Chandel, and S. Bhagwat. *Simulation of inventory policy for product with price and time dependent demand for deteriorating item*, International Journal of Modeling, Simulation, and Scientific Computing 3(1), 1-30, 2010c.

[15] J. T. Teng and C. T. Chang. *Economic production quantity model for deteriorating items with price and stock dependent demand*, Computer and Operations Research 32(2), 297-308, 2005.

[16] T. L. Urban and R. C. Baker. *Optimal ordering and pricing policies in a single-period environment with multivariate demand and market down*, European Journal of Operational Research 103, 573-583, 1997.

[17] S. P. You. *Inventory policy for products with price and time-dependent demands*, Journal of Operations Research Society 56(7), 870-873, 2005.

[18] M. Ziaee, F. Asgari, and S. J. Sadjadi. *New formulation for vendor managed inventory problem*, Trend in Applied Science Research 6, 438-450, 2011.