The Nature of Stress Pattern Due to a Sudden Movement Across a Nonplanar Buried Strike-Slip Fault In a Layered Medium

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Abstract. A layered model of the lithosphere-asthenosphere system consisting of an elastic layer overlying a viscoelastic half space is considered. A long, buried, inclined strike-slip fault is taken to be situated in the medium with a bending at the surface of seperation. The mantle convection induced a constant force on the fault resulting in a sudden movement across it. The movement on the fault changes the stress pattern in the nearby region. A mathematical model incorporating the essential features of the tectonic forces and the associated fault movement has been developed. Analytical expression for displacements, strains and stresses are obtained using suitable mathematical techniques involving integral transforms, Green's function etc. Computational work indicates that a sudden movement across the fault as significant effect on the stress accumulation in the region. The variation of shering stress with depth and distance from the fault show some iteresting features. It is expected that such features will be usefull in understanding the mechanism of eartquakes processes during the aseismic period.

Key Words and Phrases: Aseismic, Elastic layer, Viscoelastic half apace, Strike-slip faults, Green's function, Stress accumulation.

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1. Introduction

The nature of ground deformation during the aseismic period in seismically active regions in between two major seismic events should be studied in depth for a better understanding of the stress accumulation pattern in the region. Such studies can be carried out by developing suitable mathematical models incorporating the essential features of the local geological structure and the eartquake faults situated in the region. In the present paper, the lithosphere-asthenosphere system has been taken to be represented by a layered

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model consisting of an elastic layer of finite depth overlying and is welded contact with a viscoelastic half space. In most of the earlier studies (e.g.Steketee,J.A.[32],Maruyama,T.[17], Rybicki,K. [28], Sato,R.[29, 30],Chinnery,M.A., [4, 5, 6, 7, 8, 9],Rani,S.,Sing,S.[26, 27], Mukhopadhyay,et.al. [18, 19, 20, 21, 22, 23, 24],Brink,U.S.et.al., [1],Martin,F.L.,[15],Oglesby, D.,[25], Ide,S., [11], Fowler,A.C.,[10]) the faults are usually taken as single planar fault, but in reality the earthquake faults are not planar, but consisit of a number of planar segments. In view of this a long, buried, strike-slip fault F having two adjacent planar parts F_1 and F_2 is taken to be situated in the model with the part F_1 in the layer and F_2 in the half space with a common line of joint on the surface of separation. Tectonic forces, primarily due to mantle convection, results in accumulation of shear stress in the vicinity of the fault, which, in turn lead to a sudden movement across either F_1 and / or F_2 causing earthquakes, when the accumulated stress exceeds the frictional and cohesive forces across the fault. The sudden movement across the fault induces significant in changes the stress accumulation pattern in the region. The present paper has stressed upon these aspect in detail and tried to correlate such changes with the prediction of next major seismic event.

2. Formulation

We consider a simple theoretical model of the lithosphere-asthenosphere system with a locked buried and long, non-planar strike-slip fault consisting of two planar parts F_1 and F_2 with edges parallel to the free surface. The first part F_1 which is situated in elastic layer of depth H inclined at an angle θ_1 with horizontal and second part F_2 which is situated in viscoelastic half-space inclined at an angle θ_2 with horizontal.

We introduce a rectangular Cartesian coordinate system (y_1, y_2, y_3) with origin o at free surface with the plane free surface $y_3 = 0$ and y_1 axis is taken along the straight line on the free surface which is parallel to upper edge of the fault. The boundary between layer and half-space is given by $y_3 = H$.

For convenience of the analysis we introduce another two rectangular system of Cartesian coordinates (y_1', y_2', y_3') and (y_1'', y_2'', y_3'') associate with the parts F_1 and F_2 of the fault respectively with origin at $o'(0, 0, d_1)$ for first and $o''(0, l_1 cos \ \theta_1, d_1 + l_1 sin \ \theta_1)$ for second part.

The plane of first part F_1 of the fault is given by the plane $y_2' = 0$ and the plane of second part F_2 of the fault is given by the plane $y_2'' = 0$. With this choice of axes the layered medium occupies the region $0 \leq y_3 \leq H$ and the half space occupies the region $y_3 \geq H$. While the fault is given by ($F_1 : y_2' = 0, 0 \leq y_3' \leq l_1 \ and \ F_2 : y_2'' = 0, 0 \leq y_3'' \leq l_2$). The relations between $(y_1, y_2, y_3), (y_1', y_2', y_3')$ and (y_1'', y_2'', y_3'') are given by :

$$y_1 = y_1'$$
$$y_2 = y_2' sin \ \theta_1 + y_3' cos \ \theta_1$$
$$y_3 = -y_2' cos \ \theta_1 + y_3' sin \ \theta_1 + d_1$$

and

$$y_1 = y_1''$$
$$y_2 = y_2'' sin\ \theta_2 + y_3'' cos\ \theta_2 + l_1 cos\ \theta_1$$
$$y_3 = -y_2'' cos\ \theta_2 + y_3'' sin\ \theta_2 + l_1 sin\ \theta_1$$

A section of the theoretical model by the plane $y_1 = 0$ has been shown in the Fig : 1 in which the coordinate axes (y_2, y_3), (y_2', y_3') and (y_2'', y_3'') have also been identified.
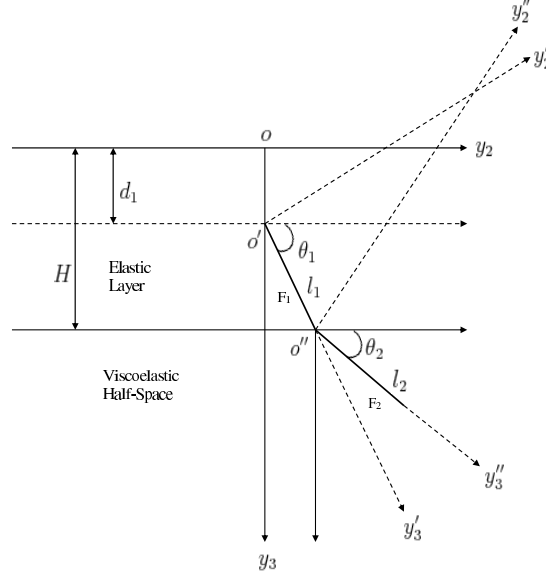

Figure 1: Section of the model by the plane $y_1 = 0$ and coordinate system.

We assume that the length of the faults are large compared to their depths, so that the displacements, stresses and strains may be taken to be independent of y_1 and dependent on y_2, y_3 and time t. With this assumption, the components of displacement, stress and strain u_1, (τ_{12}, τ_{13}) and (e_{12}, e_{13}) in the elastic layer and u_1', (τ_{12}', τ_{13}') and (e_{12}', e_{13}') in the viscoelastic half space are found to be associated with the strike-slip movement of the fault. The material of the half-space is assumed to be linearly viscoelastic and of the Maxwell type.

We start with a situation when the model is in a quasi-static, aseismic state and is undergoing slow, aseismic deformations, with a shear stress τ_∞ in the model far away from the fault maintained by some tectonic forces arising possibly due to mantle convection and / or other geological changes. We measure the time t from a suitable instant in the aseismic state of the model, before any fault movement occurs.

For the elastic layer, the constitutive equations are taken to be

$$\left. \begin{array}{l} \tau_{12} = \mu_1 \frac{\partial u_1}{\partial y_2} \\ \tau_{13} = \mu_1 \frac{\partial u_1}{\partial y_3} \end{array} \right\} \qquad (1)$$

$$0 \le y_3 \le H, \ -\infty < y_2 < \infty$$

3

where μ_1 is the rigidity of the elastic layer.

For the viscoelastic half-space of Maxwell type, the constitutive equations are

$$\left.\begin{array}{c} (\frac{1}{\eta} + \frac{1}{\mu_2}\frac{\partial}{\partial t})\tau'_{12} = \frac{\partial^2 u'_1}{\partial t \partial y_2} \\[2mm] (\frac{1}{\eta} + \frac{1}{\mu_2}\frac{\partial}{\partial t})\tau'_{13} = \frac{\partial^2 u'_1}{\partial t \partial y_3} \end{array}\right\} \qquad (2)$$

$$y_3 \geq H, \ t \geq 0, \ -\infty < y_2 < \infty$$

where μ_2 and η are the effective rigidity and viscosity respectively. We consider quasi-static aseismic deformation of the system when the inertial terms in the stress equations of motion are small and can be neglected, as explained by Mukhopadhyay et.al.,[21]. For such aseismic deformation the stresses satisfy the relations

$$\left.\begin{array}{c} \frac{\partial \tau_{12}}{\partial y_2} + \frac{\partial \tau_{13}}{\partial y_3} = 0 \ (0 \leq y_3 \leq H) \\[2mm] \frac{\partial \tau'_{12}}{\partial y_2} + \frac{\partial \tau'_{13}}{\partial y_3} = 0 \ (y_3 \geq H) \end{array}\right\} \qquad (3)$$

From (1),(2) and (3) we find that

$$\frac{\partial}{\partial t}(\nabla^2 u_1) = 0$$

and

$$\frac{\partial}{\partial t}(\nabla^2 u'_1) = 0$$

which are satisfied if

$$\left.\begin{array}{c} \nabla^2 u_1 = 0 \ (0 \leq y_3 \leq H) \\ \nabla^2 u'_1 = 0 \ (y_3 \geq H) \end{array}\right\} \qquad (4)$$

$$(t \geq 0, -\infty < y_2 < \infty)$$

We assusme that the upper surface of the layer is stress-free and the upper layer is in welded contact with the half-space. Then the displacements and stresses would satify the following boundary conditions :

$$\left.\begin{array}{l} \tau_{13} = 0 \ at \ y_3 = 0 \\ \tau_{13} = \tau'_{13} \ at \ y_3 = H \\ u_1 = u'_1 \ at \ y_3 = H \\ \tau'_{13} \to 0 \ as \ y_3 \to \infty \end{array}\right\} \qquad (5)$$

$$(t \geq 0, -\infty < y_2 < \infty)$$

We assume that tectonic forces result in a shear strain far away from the fault which may changes with time. We then have the following boundary conditions :

$$\left.\begin{array}{l} e_{12} \to (e_{12})_{0\infty} + g(t) \\ e'_{12} \to (e'_{12})_{0\infty} + g(t) \end{array}\right\} \qquad (6)$$

4

$$as \ |y_2| \to \infty \ for \ t \ge 0$$

where

$$(e_{12})_{0\infty} = \lim_{|y_2| \to \infty} (e_{12})_0$$

$$(e'_{12})_{0\infty} = \lim_{|y_2| \to \infty} (e'_{12})_0$$

where $(e_{12})_0$, $(e'_{12})_0$ are the values of e_{12} and e'_{12} at $t = 0$ and $g(t)$ is a continuous function of t such that $g(0) = 0$. Same $g(t)$ is taken for both e_{12} and e'_{12}, since the media are in welded contact.

2.1. Initial conditions :

We measure time t from a suitable instant when the model is in aseismic state and there is no seismic disturbance in it. $(u_1)_0$, $(u'_1)_0$ $(\tau_{12})_0$, $(\tau'_{12})_0$, $(\tau_{13})_0$, $(\tau'_{13})_0$, $(e_{12})_0$, $(e'_{12})_0$, $(e_{13})_0$, $(e'_{13})_0$ are values of u, u', τ_{12}, τ'_{12}, τ_{13}, τ'_{13}, e_{12}, e'_{12}, e_{13}, e'_{13} at time $t = 0$ and they satisfy the relations (1)–(6).

2.2. Displacements, stressses and strains in the absence of fault movement :

We start with the situation when the system is in aseismic state. In this case displacements, stresses and strains are continuous throughout the system and all the equations and boundary conditions given in (1)–(6) are valid. To obtain the solutions for displacements, stresses and strains in absence of any fault movement, we take Laplace transformations of (1)–(6) with respect to time t. This gives us a boundary value problem which can be solved easily. Finally, on inverting the Laplace transforms we get the following solutions

$$\left.\begin{array}{l} u_1 = (u_1)_0 + y_2 g(t) \\ \tau_{12} = (\tau_{12})_0 + \mu_1 g(t) \\ \tau_{13} = (\tau_{13})_0 \\ e_{12} = (e_{12})_0 + g(t) \\ \tau_{12} = (\tau_{12})_0 + \mu_1 sin \ \theta g(t) \end{array}\right\} \quad (7)$$

$$\left.\begin{array}{l} u'_1 = (u'_1)_0 + y_2 g(t) \\ \tau'_{12} = (\tau'_{12})_0 exp(-\frac{\mu_2 t}{\eta}) + \mu_2 \int_0^t g_1(\tau) exp\{-\frac{\mu_2(t-\tau)}{\eta}\} d\tau \\ \tau'_{13} = (\tau'_{13})_0 exp(-\frac{\mu_2 t}{\eta}) \\ e'_{12} = (e'_{12})_0 + g(t) \end{array}\right\} \quad (8)$$

where $g_1(t) = \frac{d}{dt}\{g(t)\}$

We assume that the strain e_{12} and e'_{12} gradually increases under the action of τ_∞. So that both $g(t)$ and $g_1(t)$ are taken to be increasing functions of time t. From (7) and (8) we find that both τ_{12} and τ'_{12} increase with time. When the accumulated stresses exceeds some threshold values the fault parts F_1 and F_2 undergo sudden movement resulting in an eartquake.

2.3. Displacements, stresses and strains after the restoration of aseismic state folowing a sudden strike-slip movement across the fault part F :

The sudden movement across F has two constitutive parts– movement across F_1 and at the same time a movement across F_2. We first consider the effect of sudden movement across F_1. It is to be noted that due to the sudden fault movement across the fault F, the accumulated stress will be released at least to some extent and the fault becomes locked again when the shear stress near the fault has sufficiently been released. For a comparatively short period of time, during and aftter the sudden fault movement when the seismic disturbances generated by this fault movement are still present in the vicinity of the fault, the inertial forces are not small and can not be neglected. We leave out this short period of time, during and immediately after sudden fault movement and consider the model after the restoration of the aseismic state, which happens when the seismic disturbances near the fault gradually disappear. We shall determine the displacements, stresses and strains during the second phase of aseismic state with respect to new time origin $t = 0$, denoting the instant at which this aseismic state has been restored in the system after sudden fault movement.

We note that, for period $t \geq 0$ (corresponding to the new phase of aseismic state of the model re-established after the sudden fault movement), the inertial forces again become very small and are therefore neglected, so that all the equations from (1)–(6) are valid in this case also. The displacements, stresses and strains are continuous everywhere except for the fault F_1, F_2. The displacement component u_1 has a discontinuity which characterises the sudden fault movement across the fault F_1 given by the following conditions :

$$[u_1] = U_1 f_1(y_3') \; across \; F_1 \; (y_2' = 0, \; 0 \leq y_3' \leq l_1, \; t \geq 0) \qquad (9)$$

where $[u_1]$ is the discontinuity of u_1 across F_1 defined as

$$[u_1] = \lim_{y_2' \to 0^+} (u_1) - \lim_{y_2' \to 0^-} (u_1)$$

and $f_1(y_3')$ is a continuous function of y_3' giving the dependence of the relative displacement across F_1 on the depth along the fault part F_1 and U_1 is a constant, independent of y_2', y_3'. Similarly, for sudden movement across F_2, we have :

$$[u_1'] = U_2 f_2(y_3'') \; across \; F_2 \; (y_2'' = 0, \; 0 \leq y_3'' \leq l_2, \; t \geq 0) \qquad (10)$$

where $[u_1']$ is the discontinuity of u_1' across F_2 defined as

$$[u_1'] = \lim_{y_2'' \to 0^+} (u_1') - \lim_{y_2'' \to 0^-} (u_1')$$

and $f_2(y_3'')$ is a continuous function of y_3'' giving the dependence of the relative displacement across F_2 on the depth along the fault part F_2 and U_2 is a constant, independent of y_2'', y_3''.

The stresses and strains τ_{12}, τ_{13}, τ_{12}', τ_{13}', e_{12}, e_{12}' are continuous everywhere in the model.

We try to obtain displacements and stresses for $t \geq 0$ (with new time origin) due to movement across F_1 in the form

$$\left.\begin{array}{l} u_1 = (u_1)_1 + (u_1)_2 \\ \tau_{12} = (\tau_{12})_1 + (\tau_{12})_2 \\ \tau_{13} = (\tau_{13})_1 + (\tau_{13})_2 \\ u_1' = (u_1')_1 + (u_1')_2 \\ \tau_{12}' = (\tau_{12}')_1 + (\tau_{12}')_2 \\ \tau_{13}' = (\tau_{13}')_1 + (\tau_{13}')_2 \end{array}\right\} \qquad (11)$$

where $(u_1)_1$, $(\tau_{12})_1$, ..., $(\tau_{13}')_1$ satisfy relations (1)–(8) are continuous everywhere in the model. The solutions for $(u_1)_1$, $(\tau_{12})_1$, ..., $(\tau_{13}')_1$ are therefore given by

$$\left.\begin{array}{l} (u_1)_1 = (u_1)_p + y_2 g(t) \\ (\tau_{12})_1 = (\tau_{12})_p + \mu_1 g(t) \\ (\tau_{13})_1 = (\tau_{13})_p \end{array}\right\} \qquad (12)$$

$$\left.\begin{array}{l} (u_1')_1 = (u_1')_p + y_2 g(t) \\ (\tau_{12}')_1 = (\tau_{12}')_p exp(-\frac{\mu_2 t}{\eta}) + \mu_2 \int\limits_0^t g_1(\tau) exp\{-\frac{\mu_2(t-\tau)}{\eta}\}d\tau \\ (\tau_{13}')_1 = (\tau_{13}')_p exp(-\frac{\mu_2 t}{\eta}) \end{array}\right\} \qquad (13)$$

where $(u_1)_p$, $(u_1')_p$, ..., $(\tau_{13}')_p$ are the values of $(u_1)_1$, $(u_1')_1$, ..., $(\tau_{13}')_1$ at time $t = 0$ (i.e. the new time origin) satisfying all conditions from (1)–(8) and $g_1(t) = \frac{\mathrm{d}}{\mathrm{dt}} g(t)$ and $(u_1)_2$, $(u_1')_2$, ..., $(\tau_{13}')_2$ satisfy the relations (1)–(5) and the dislocation condition (9) together with the following conditions :

$$\left.\begin{array}{l} (e_{12}) \to 0 \ as \ |y_2| \to \infty \ for \ t \geq 0 \\ (e_{12}') \to 0 \ as \ |y_2| \to \infty \ for \ t \geq 0 \end{array}\right\} \qquad (14)$$

Thus $(u_1)_2$, $(u_1')_2$, ..., $(\tau_{13}')_2$ satisfy the following relations

$$\left.\begin{array}{l} (\tau_{12})_2 = \mu_1 \frac{\partial}{\partial y_2}(u_1)_2 \\ \\ (\tau_{13})_2 = \mu_1 \frac{\partial}{\partial y_3}(u_1)_2 \end{array}\right\} \qquad (15)$$

$$-\infty < y_2 < \infty, \ 0 \leq y_3 \leq H, \ t \geq 0$$

$$\left.\begin{array}{l} (\frac{1}{\eta} + \frac{1}{\mu_2}\frac{\partial}{\partial t})(\tau_{12}')_2 = \frac{\partial^2 (u_1')_2}{\partial t \partial y_2} \\ \\ (\frac{1}{\eta} + \frac{1}{\mu_2}\frac{\partial}{\partial t})(\tau_{13}')_2 = \frac{\partial^2 (u_1')_2}{\partial t \partial y_3} \end{array}\right\} \qquad (16)$$

$$-\infty < y_2 < \infty, \ y_3 \geq H, \ t \geq 0$$

$$\left.\begin{array}{l} \frac{\partial}{\partial y_2}(\tau_{12})_2 + \frac{\partial}{\partial y_3}(\tau_{13})_2 = 0, \ 0 \leq y_3 \leq H \\ \\ \frac{\partial}{\partial y_2}(\tau_{12}')_2 + \frac{\partial}{\partial y_3}(\tau_{13}')_2 = 0, \ y_3 \geq H \end{array}\right\} \qquad (17)$$

7

$$-\infty < y_2 < \infty, \ t \geq 0$$

$$\left.\begin{array}{l} \nabla^2 (u_1)_2 = 0, \ 0 \leq y_3 \leq H \\ \nabla^2 (u_1')_2 = 0, \ y_3 \geq H \end{array}\right\} \qquad (18)$$

$$-\infty < y_2 < \infty, \ t \geq 0$$

$$\left.\begin{array}{l} (\tau_{13})_2 = 0 \ at \ y_3 = 0 \\ (\tau_{13})_2 = (\tau_{13}')_2 \ at \ y_3 = H \\ (u_1)_2 = (u_1')_2 \ at \ y_3 = H \\ (\tau_{13}')_2 \to 0 \ as \ y_3 \to \infty \end{array}\right\} \qquad (19)$$

$$-\infty < y_2 < \infty, \ t \geq 0$$

$$\left.\begin{array}{l} (e_{12})_2 \to 0 \\ (e_{12}')_2 \to 0 \end{array}\right\} \qquad (20)$$

$$as \ |y_2| \to \infty, \ t \geq 0$$

and

$$[(u_1)_2] = U_1 f_1(y_3') \ across \ F_1 : (y_2' = 0, \ 0 \leq y_3' \leq l_1, \ t \geq 0) \ \} \qquad (21)$$

To obtain the solutions for $(u_1)_2, (u_1')_2, \ldots, (\tau_{13}')_2$ satisfying the above relations, we take Laplace transforms of these relations with respect to t. The resulting boundary value problem involving $(\overline{u}_1)_2, (\overline{u_1'})_2, \ldots, (\overline{\tau_{13}'})_2$, which are the Laplace transforms of $(u_1)_2, (u_1')_2, \ldots, (\tau_{13}')_2$ respectively with respect to t, can be solved by using a modified form of Green's function technique developed by Maruyama,T.,[17] and Rybicki,K.,[28] as explained in APPENDIX. On taking inverse Laplace transforms, we obtain the complete solutions for $(u_1)_2, (u_1')_2, \ldots, (\tau_{13}')_2$ for $t \geq 0$. Finally we obtain the complete solutions $u_1, u_1', \ldots, \tau_{13}'$ from (11) as follows :

$$\left.\begin{array}{l} u_1(y_2, y_3, t) = (u_1)_p + y_2 g(t) + \frac{U_1}{2\pi} \psi_1(y_2, y_3, t) \\[4pt] e_{12}(y_2, y_3, t) = (e_{12})_p + g(t) + \frac{U_1}{2\pi} \psi_2(y_2, y_3, t) \\[4pt] \tau_{12}(y_2, y_3, t) = (\tau_{12})_p + \mu_1 g(t) + \frac{\mu_1 U_1}{2\pi} \psi_2(y_2, y_3, t) \\[4pt] \tau_{13}(y_2, y_3, t) = (\tau_{13})_p + \frac{\mu_1 U_1}{2\pi} \psi_3(y_2, y_3, t) \\[4pt] \tau_{1'2'} = \tau_{12} \sin \theta_1 - \tau_{13} \cos \theta_1 \\[4pt] u_1'(y_2, y_3, t) = (u_1')_p + y_2 g(t) + \frac{U_1}{\pi} \phi_1(y_2, y_3, t) \\[4pt] \tau_{12}'(y_2, y_3, t) = (\tau_{12}')_p \exp(-\frac{\mu_2 t}{\eta}) + \mu_2 \int_0^t g_1(\tau) \exp\{-\frac{\mu_2(t-\tau)}{\eta}\} d\tau + \frac{U_1}{\pi} \phi_2(y_2, y_3, t) \\[4pt] \tau_{13}'(y_2, y_3, t) = (\tau_{13}')_p \exp(-\frac{\mu_2 t}{\eta}) + \frac{U_1}{\pi} \phi_3(y_2, y_3, t) \\[4pt] \tau_{1''2''}' = \tau_{12}' \sin \theta_2 - \tau_{13}' \cos \theta_2 \end{array}\right\} \qquad (22)$$

where ψ_1, ψ_2, ψ_3 and ϕ_1, ϕ_2, ϕ_3 are given in APPENDIX.

Due to movement of fault part F_2 the displacement component $(u_1)_3$, stress components $(\tau_{12})_3, \ (\tau_{13})_3$ of elastic layer satisfies equations (1)–(5) and displacement component $(u_1')_3$,

stress components $(\tau'_{12})_3$, $(\tau'_{13})_3$ of viscoelastic half-space satisfies also equations (1)–(5) and also they satisfies dislocations condition (9) together with the conditions

$$(e_{12})_3 \to 0 \; as \; |y_2| \to \infty \; for \; t \geq 0$$
$$(e'_{12})_3 \to 0 \; as \; |y_2| \to \infty \; for \; t \geq 0$$

Using the similar method we get the solution as

$$
\left.
\begin{aligned}
(u_1)_3(y_2, y_3, t) &= \tfrac{U_2}{\pi} \psi'_1(y_2, y_3, t) \\[2mm]
(e_{12})_3(y_2, y_3, t) &= \tfrac{U_2}{\pi} \psi'_2(y_2, y_3, t) \\[2mm]
(\tau_{12})_3(y_2, y_3, t) &= \tfrac{\mu_1 U_2}{\pi} \psi'_2(y_2, y_3, t) \\[2mm]
(\tau_{13})_3(y_2, y_3, t) &= \tfrac{\mu_1 U_2}{\pi} \psi'_3(y_2, y_3, t) \\[2mm]
(u'_1)_3(y_2, y_3, t) &= \tfrac{U_2}{2\pi} \phi'_1(y_2, y_3, t) \\[2mm]
(\tau'_{12})_3(y_2, y_3, t) &= \tfrac{U_2}{2\pi} \phi'_2(y_2, y_3, t) \\[2mm]
(\tau'_{13})_3(y_2, y_3, t) &= \tfrac{U_2}{2\pi} \phi'_3(y_2, y_3, t)
\end{aligned}
\right\} \qquad (23)
$$

where the explicit form of functions ψ'_1, ψ'_2, ψ'_3, ϕ'_1, ϕ'_2, ϕ'_3 are given by (A30), (A31), (A32), (A34), (A35), (A36) respectively in APPENDIX.

The complete solution is given by

$$
\left.
\begin{aligned}
u_1 &= (u_1)_1 + (u_1)_2 + (u_1)_3 \\
e_{12} &= (e_{12})_1 + (e_{12})_2 + (e_{12})_3 \\
\tau_{12} &= (\tau_{12})_1 + (\tau_{12})_2 + (\tau_{12})_3 \\
\tau_{13} &= (\tau_{13})_1 + (\tau_{13})_2 + (\tau_{13})_3 \\
u'_1 &= (u'_1)_1 + (u'_1)_2 + (u'_1)_3 \\
\tau'_{12} &= (\tau'_{12})_1 + (\tau'_{12})_2 + (\tau'_{12})_3 \\
\tau'_{13} &= (\tau'_{13})_1 + (\tau'_{13})_2 + (\tau'_{13})_3
\end{aligned}
\right\} \qquad (24)
$$

It is found that for sudden movement the displacements, stresses and strains will be finite and single valued everywhere in the model, if the following conditions are satisfied :
For $f_1(y'_3)$:

(i) $f_1(y'_3), f'_1(y'_3)$ are continuous functions of y'_3 for $0 \leq y'_3 \leq l_1$

(ii) $f'_1(0) = 0$.

(iii) $f''_1(y'_3)$ is continuous in $0 \leq y'_3 \leq l_1$ except for a finite number of points of finite discontinuity in $0 \leq y'_3 \leq l_1$ or, $f''_1(y'_3)$ is continuous in $0 < y'_3 < l_1$ and there exist real constants $m, n < 1$ such that $(y'_3)^m f''_1(y'_3) \to 0$ or to a finite limit as $y'_3 \to 0^+$ and that $(l_1 - y'_3)^n f''(y'_3) \to 0$ or to a finite limit as $y'_3 \to l_1^{-0}$.

9

For $f_2(y_3'')$:

(i) $f_2(y_3''), f_2'(y_3'')$ are continuous functions of y_3'' for $0 \leq y_3'' \leq l_2$.

(ii) $f_2(l_2) = 0$ *and* $f_2'(0) = f_2'(l_2) = 0$.

(iii) Either $f_2''(y_3'')$ is continuous in $0 \leq y_3'' \leq l_2$ or, $f_2''(y_3'')$ is continuous in $0 \leq y_3'' \leq l_2$ except for a finite number of points of finite discontinuity in $0 \leq y_3'' \leq l_2$ or, $f_2''(y_3'')$ is continuous in $0 < y_3'' < l_2$ and there exist real constants $m, n < 1$ such that $(l_2 - y_3'')^m f_2''(y_3'') \to 0$ or to a finite limit as $y_3'' \to l_2^{-0}$ and $(y_3'')^n f_2''(y_3'') \to 0$ or to a finite limit as $y_3'' \to 0^{+0}$.

3. Results and Discussions

The following values of the model parameters are taken for numerical computations : $l_1 = 10$ km., $l_2 = 12$ km. are length of the fault parts F_1 and F_2 respectively.

$H =$ width of the elastic layer$= 40$ km, representing the upper part of the lithosphere (the crust).

$\mu_1 =$ the rigidity of the elastic layer$= 3.0 \times 10^{11}$ dyne/sq.cm.

$\mu_2 =$ the effective rigidity of the viscoelastic half space representing the asthenosphere (up to depth of about 600 km)$= 3.78 \times 10^{11}$ dyne/sq.cm.

$\eta =$ the viscosity of the half space $= 3 \times 10^{21}$ poise.

$U_1 = 40$ cm and $U_2 = 40$ cm, are slip across the fault F_1 and F_2 respectively.

The above values are taken from different books and research publications (e.g. Cathles,[2], Chift,P.et.al.[3], Karato,S.[13]).

We carried out the numerical computations with the following choice of $f_1(y_3')$ and $f_2(y_3'')$: $f_1(y_3') = 1 - (\frac{y_3'}{l_1})^2 + \frac{1}{2}(\frac{y_3'}{l_1})^3$ and $f_2(y_3'') = \frac{(y_3''^2 - l_2^2)^2}{2l_2^4}$. The depth dependence of f_1 and f_2 are so chose in a way that the continuity at the common edge be maintained i.e.$f_1(at \ y_3' = l_1) = f_2(at \ y_3'' = l_2) =$ k is chosen as $\frac{1}{2}$ (may be taken otherewise). This continuity condition however violated the conditions stated earlier for bounded stress even at the common edge. However, stress very close to this common edge are found to be bounded.

We compute the following quantities :

(i) Additional surface displacement due to fault movement

W$= [u_1 - (u_1)_1 - y_2 \ g(t)]_{y_3=0}$ at $t = 0$, just after the commencement of the fault creep.

$= \frac{U_1}{2\pi}\psi_1 + \frac{U_2}{\pi}\psi_1'$ for different values of θ_1 and θ_2 (fig. 2 and 3).

(ii) Surface shear strain (R_s) given by

$(R_s)=[e_{12} - (e_{12})_p - g(t)]_{y_3} = 0$ at $t = 0$, just after the commencement of the fault creep.

$=\frac{U_1}{2\pi}\psi_2 + \frac{U_2}{\pi}\psi_2'$ (fig. 4 and 5).

10

(iii) Variation of shear stress τ_{12} with depth within the elastic layer and viscoelastic half space are respectively given by

$$(\tau_{12}) = \tau_{12} - (\tau_{12})_p - \mu_1 \ g(t)$$
$$= \frac{\mu_1 U_1}{2\pi}\psi_2 + \frac{\mu_1 U_2}{\pi}\psi_2' \text{ (in fig. 6)}$$

and

$$(\tau_{12})' = \tau_{12}' - (\tau_{12}')_p e^{-\frac{\mu_2}{\eta}t} + \mu_2 \int_0^t g_1(\tau)exp\{-\frac{\mu_2}{\eta}(t-\tau)\}d\tau$$
$$= \frac{U_1}{\pi}\phi_2 + \frac{U_2}{2\pi}\phi_2' \text{ (in fig. 7)}$$

(iv) Variation of shear stress τ_{13} with depth within the elastic layer and viscoelastic half space are respectively given by

$$(\tau_{13}) = \tau_{13} - (\tau_{13})_p$$
$$= \frac{\mu_1 U_1}{2\pi}\psi_3 + \frac{\mu_1 U_2}{\pi}\psi_3' \text{ (in fig. 8)}$$

and

$$(\tau_{13})' = \tau_{13}' - (\tau_{13}')_p e^{-\frac{\mu_2}{\eta}t}$$
$$= \frac{U_1}{\pi}\phi_3 + \frac{U_2}{2\pi}\phi_3' \text{ (in fig. 9)}$$

(v) The stress pattern changes due to the presence of an elastic layer over lying the viscoelastic half space when compared with the half space model. (T_{12}) represent these changes due to presence of the layer given by :

$T_{12} = sin \ \theta_1(\frac{\mu_1 U_1}{2\pi} \times series \ part \ of \ \psi_2 + \frac{\mu_1 U_2}{\pi} \times series \ part \ of \ \psi_2') - cos \ \theta_1(\frac{\mu_1 U_1}{2\pi} \times series \ part \ of \ \psi_3 + \frac{\mu_1 U_2}{\pi} \times series \ part \ of \ \psi_3')$ (in fig. 10)

Fig. 2 and 3 show the surface displacements due to the fault movements across F_1 and F_2 after one year with different combinations of θ_1 and θ_2. For $\theta_1 = \theta_2 = \frac{\pi}{2}$, the surface displacement curve is anti-symmetric. In each case there are regions of displacements is opposite directions with one maximum in each direction. The magnitude of surface displacements is of the order of (-2) cm to (+3) cm, depending on θ_1 and θ_2. As we move away from the fault $|W| \to 0$ as expected.

Fig. 4 and 5 show the change in surface shear strain immediately after the sudden movement across F_1 and F_2, for different values of θ_1 and θ_2. It is found from these figures that movement across the part F_1 is more pronounced on the change of surface shear shear strain. In both the cases, the magnitude of strain is of the order of 10^{-7}, which is good agrement with the observed value. The changes in shear strain is found to be maximum near the faults and gradually die out as we move away from the fault.

Fig. 6 indicates the changes in shear stress τ_{12} with depth in the layer 10 years after the re-establishment of aseismic state following the sudden movement across F_1 and F_2 along a vertical at which $y_2 = 15$ km. It is found that there is a region of marginal accumulation upto a depth of about 15 km and thereafter the accumulated stresses reduce

11

to some extent due to the sudden movement across the fault. The magnitude of stress enhancement/ reduction depends on the angles θ_1 and θ_2.

Fig. 7 show the variations of shear stress τ_{12} with depth in the half space due to fault movement along a vertical through $y_2 = 15$ km one year after sudden movement. For vertical fault ($\theta_1 = \theta_2 = \frac{\pi}{2}$), the changes are negligbily small. It is found that in most of the region in the half space there are stress reduction up to a depth of about 70 km from the the free surface. The magnitude of stress reduction however does not exceed 45 bars. The reduction attains maximum values at a depth of about (30-45) km, that is just below the lower edge of the fault segment F_2. The effect of the fault movement dies out beyond $y_3 = 70$ km.

Fig. 8 and 9 show the changes in the stress component(τ_{13}) induced by the fault movement in the layer and in the half space respectively along vertical at which $y_2 = 15$ km one year after the re-establishment the aseimic state following the sudden movement across the fault. In the layer there are a region of stress accumulation upto a depth about 35 km followed by the narrow region of about 5 km depth where accumulated stress are reduced due to fault movement. The angles of inclination of the fault parts F_1 and F_2 do not have much influence on the changes in the stress pattern. In the layer however the entire region is a stress reducing region indicating that accumulated stress in this region will be reduced some extent due to the sudden movement of the faults. The effects become negligibly small at a depth greater than or equal to 70 km from the free surface.

A comparative studying for layered model and half space model of the liyhosphere-asthenosphere system :

Most often the liyhosphere-asthenosphere system are modelled by taking an elastic layer overlying the viscoelastic half space. An attempt has been made to identify the extent to which in stress accumulation pattern due to fault movement differ if we stress upon a single half space model instead of a layered model.

In the expression for stresses given in equation (24) the part involving infinite series are due to the presence of the elastic layer. Numerical computations show that contribution of this part remain well below the value 0.1 bar at a point for which $y_2 = 2$ km and $y_3 = 10$ km increasing very slowly with time (0.0002 bar/year(Fig. 10)). The ratio of change in magnitude of τ_{12} for half space model and layered model is found to be (400:1) indicating that a half space model quite reasonable for representing liyhosphere-asthenosphere system.
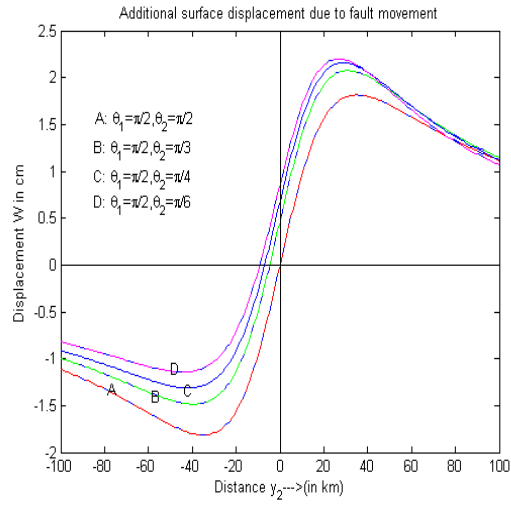
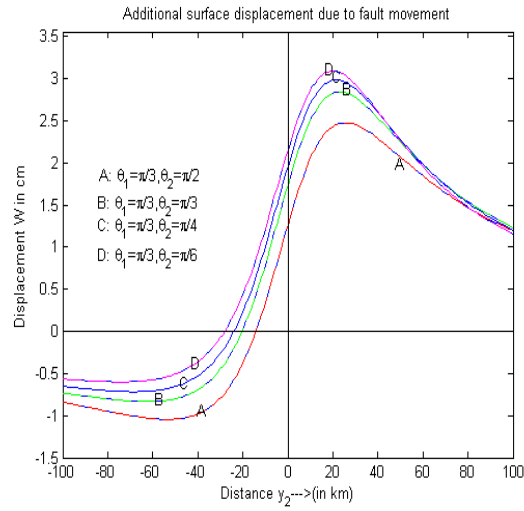Figure 2: Additional surface displacement due to fault movement.

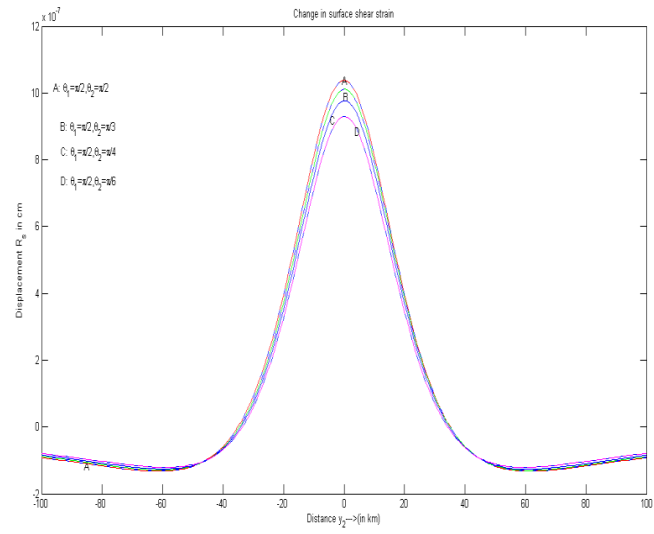Figure 3: Additional surface displacement due to fault movement.

13

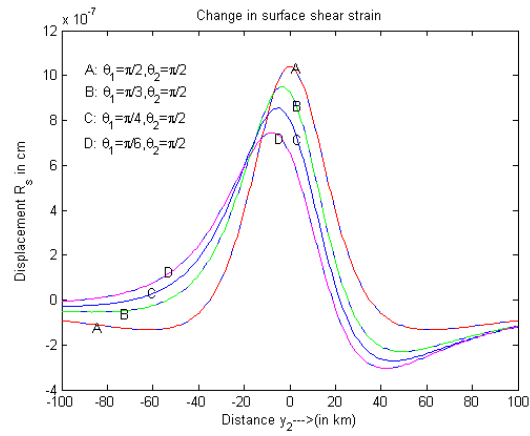Figure 4: Change in shear strain.
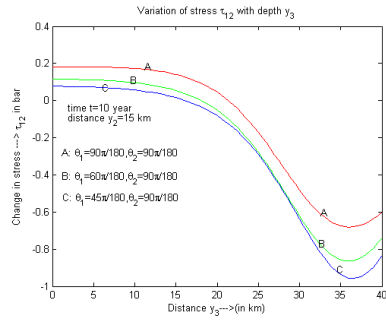

Figure 5: Change in shear strain.

14

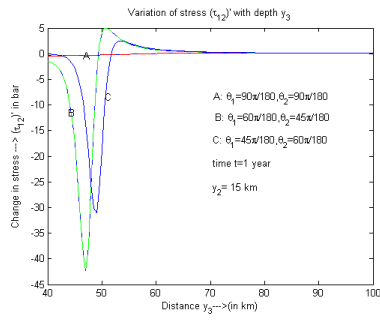Figure 6: Variation of shear stress with depth in elastic layer.

Figure 7: Variation of shear stress with depth in viscoelastic half space.
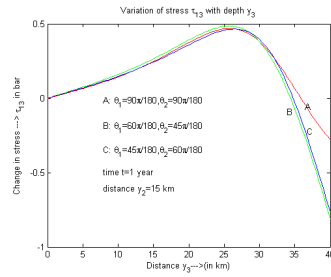

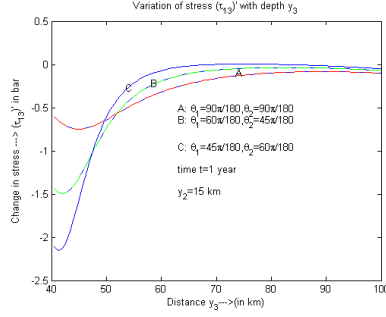Figure 8: Variation of shear stress with depth in elastic layer.

15

Figure 9: Variation of shear stress with depth in viscoelastic half space.
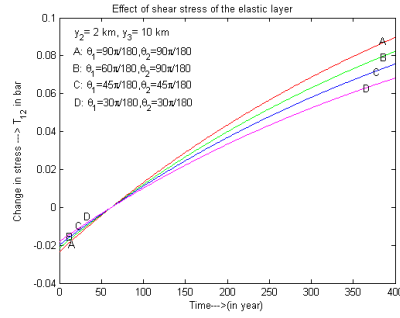

Figure 10: Changes in stress pattern doe to presence of an elastic layer overlying the viscoelastic half space.

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16

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Appendix

Displacements, stresses and strains after the restoration of aseismic state following a sudden strike-slip movement across the fault– Method of solution. Due to movement of fault part F_1, we try to obtain the solutions for displacements and stresses in the following form:

$$\left.\begin{aligned}
u_1 &= (u_1)_1 + (u_1)_2 \\
\tau_{12} &= (\tau_{12})_1 + (\tau_{12})_2 \\
\tau_{13} &= (\tau_{13})_1 + (\tau_{13})_2 \\
u_1' &= (u_1')_1 + (u_1')_2 \\
\tau_{12}' &= (\tau_{12}')_1 + (\tau_{12}')_2 \\
\tau_{13}' &= (\tau_{13}')_1 + (\tau_{13}')_2
\end{aligned}\right\} \quad (A1)$$

where $(u_1)_1$, $(\tau_{12})_1$, ..., $(\tau_{13}')_1$ are given by (12) and (13). The components of $(u_1)_2$, $(\tau_{12})_2$, ..., $(\tau_{13}')_2$ satisfying the relations (15)–(21). To obtain the solutions we take the Laplace transforms of these relations with respect to t and we get

$$\left.\begin{aligned}
(\bar{\tau}_{12})_2 &= \mu_1 \frac{\partial}{\partial y_2}(\bar{u}_1)_2 \\[2mm]
(\bar{\tau}_{13})_2 &= \mu_1 \frac{\partial}{\partial y_3}(\bar{u}_1)_2
\end{aligned}\right\} \quad (A2)$$

$$-\infty < y_2 < \infty, \ 0 \le y_3 \le H, \ t \ge 0$$

$$\left.\begin{aligned}
(\bar{\tau}_{12}')_2 &= \bar{\mu}_2 \frac{\partial}{\partial y_2}(\bar{u}_1')_2 \\[2mm]
(\bar{\tau}_{13}')_2 &= \bar{\mu}_2 \frac{\partial}{\partial y_3}(\bar{u}_1')_2
\end{aligned}\right\} \quad (A3)$$

$$-\infty < y_2 < \infty, \ y_3 \ge H, \ t \ge 0$$

where $\bar{\mu}_2 = \frac{p}{\frac{p}{\mu_2}+\frac{1}{\eta}}$

$$\left.\begin{aligned}
\frac{\partial(\bar{\tau}_{12})}{\partial y_2} + \frac{\partial(\bar{\tau}_{13})}{\partial y_3} &= 0 \ (0 \le y_3 \le H) \\[2mm]
\frac{\partial(\bar{\tau}_{12}')}{\partial y_2} + \frac{\partial(\bar{\tau}_{13}')}{\partial y_3} &= 0 \ (y_3 \ge H)
\end{aligned}\right\} \quad (A4)$$

$$-\infty < y_2 < \infty, \ y_3 \ge H, \ t \ge 0$$

$$\left.\begin{aligned}
\nabla^2(\bar{u}_1)_2 &= 0 \ (0 \le y_3 \le H) \\[2mm]
\nabla^2(\bar{u}_1')_2 &= 0 \ (y_3 \ge H)
\end{aligned}\right\} \quad (A5)$$

$$(t \ge 0, -\infty < y_2 < \infty)$$

$$\left.\begin{array}{l} (\bar{\tau}_{13})_2 = 0 \ at \ y_3 = 0 \\ (\bar{\tau}_{13})_2 = (\bar{\tau}'_{13})_2 \ at \ y_3 = H \\ (\bar{u}_1)_2 = (\bar{u}'_1)_2 \ at \ y_3 = H \\ (\bar{\tau}'_{13})_2 \to 0 \ as \ y_3 \to \infty \end{array}\right\} \qquad (A6)$$

$$-\infty < y_2 < \infty, \ t \geq 0$$

$$\left.\begin{array}{l} (\bar{e}_{12})_2 \to 0 \\ (\bar{e}'_{12})_2 \to 0 \end{array}\right\} \qquad (A7)$$

$$as \ |y_2| \to \infty, \ t \geq 0$$

and

$$\left. [(\bar{u}_1)_2] = \frac{U_1}{p} f_1(y'_3) \ across \ F_1 : (y'_2 = 0, \ 0 \leq y'_3 \leq l_1, \ t \geq 0) \right\} \qquad (A8)$$

where $\{(\bar{u}_1)_2, \ \ldots, \ (\bar{\tau}'_{13})_2\} = \int_0^\infty \{(\bar{u}_1)_2, \ \ldots, \ (\bar{\tau}'_{13})_2\} e^{-pt} dt$, p being the Laplace variable.

The boundary value problem (A1)–(A8) can be solved by using a modified Green's function technique developed by Maruyama (1966) and Rybicki (1971) and following them we get

$$(\bar{u}_1)_2(Q) = \int_{F_1} [(\bar{u}_1)_2(P)][G_{12}(P,Q)dx_3 - G_{13}(P,Q)dx_2] \qquad (A9)$$

where $Q(y_1, y_2, y_3)$ is any point in the layer and $P(x_1, x_2, x_3)$ is any point on the fault and $[(\bar{u}_1)_2(P)]$ is the magnitude of discontinuity in $(\bar{u}_1)_2$ across F_1 at P is equal to $\frac{U_1}{p} f(y'_3)$ in our model. For the half space we have

$$(\bar{u}'_1)_2(Q_1) = \int_{F_1} [(\bar{u}'_1)_2(P)][G'_{12}(P,Q_1)dx_3 - G'_{13}(P,Q_1)dx_2] \qquad (A10)$$

where $Q_1(y_1, y_2, y_3)$ is any point in the half space.

In (A9)

$$\left.\begin{array}{l} G_{13}(P,Q) = \mu_1 \frac{\partial}{\partial x_3} G_1(P,Q) \\[2mm] G_{12}(P,Q) = \mu_1 \frac{\partial}{\partial x_2} G_1(P,Q) \end{array}\right\} \qquad (A11)$$

where

$$\left.\begin{array}{l} G_1(P,Q) = -\frac{1}{2\pi\mu_1}[\ln \sqrt{(x_2 - y_2)^2 + (x_3 - y_3)^2} + \\ \quad \ln \sqrt{(x_2 - y_2)^2 + (x_3 + y_3)^2} + \\ \quad \sum_{n=1}^{\infty} (\frac{\mu_1 - \bar{\mu}_2}{\mu_1 + \bar{\mu}_2})^m \{\ln \sqrt{(x_2 - y_2)^2 + (x_3 - 2mH - y_3)^2} + \\ \qquad \ln \sqrt{(x_2 - y_2)^2 + (x_3 - 2mH + y_3)^2} + \\ \qquad \ln \sqrt{(x_2 - y_2)^2 + (x_3 + 2mH - y_3)^2} + \\ \qquad \ln \sqrt{(x_2 - y_2)^2 + (x_3 + 2mH + y_3)^2}\} \end{array}\right\} \qquad (A12)$$

Now $P(x_1, x_2, x_3)$ being a point on the fault F_1, $0 \le x_2 \le l_1 \cos \theta_1$, $0 \le x_3 \le l_1 \sin \theta_1$ and $x_2 = x_3 \cot \theta_1$. A change in coordinate axes from (x_1, x_2, x_3) to (ξ_1', ξ_2', ξ_3') connected by the relations $x_1 = \xi_1'$, $x_2 = \xi_2' \sin \theta_1 + \xi_3' \cos \theta_1$ and $x_3 = -\xi_2' \cos \theta_1 + \xi_3' \sin \theta_1$, is intruduced so that $\xi_2' = 0$ and $0 \le \xi_3' \le l_1$ on F_1.

Thus we get,

$$(\bar{u}_1)_2(Q) = \frac{U_1}{2\pi p} \int_0^{l_1} f(\xi_3') \Big[\frac{y_2 \sin \theta_1 - (y_3 - d_1)\cos \theta_1}{A_1} +$$
$$\frac{y_2 \sin \theta_1 + (y_3 + d_1)\cos \theta_1}{A_2} +$$
$$\sum_{m=1}^{\infty} \Big(\frac{\mu_1 - \bar{\mu}_2}{\mu_1 + \bar{\mu}_2} \Big)^m \Big\{ \frac{y_2 \sin \theta_1 - 2mH\cos \theta_1 - (y_3 - d_1)\cos \theta_1}{A_3} +$$
$$\frac{y_2 \sin \theta_1 - 2mH\cos \theta_1 + (y_3 + d_1)\cos \theta_1}{A_4} +$$
$$\frac{y_2 \sin \theta_1 + 2mH\cos \theta_1 - (y_3 - d_1)\cos \theta_1}{A_5} +$$
$$\frac{y_2 \sin \theta_1 + 2mH\cos \theta_1 + (y_3 + d_1)\cos \theta_1}{A_5} \Big\} \Big] d\xi_3'$$

where

$$A_1 = \xi_3'^2 - 2\xi_3'\{y_2 \cos \theta_1 + (y_3 - d_1)\sin \theta_1\} + y_2^2 + (y_3 - d_1)^2$$
$$A_2 = \xi_3'^2 - 2\xi_3'\{y_2 \cos \theta_1 - (y_3 - d_1)\sin \theta_1\} + y_2^2 + (y_3 - d_1)^2$$
$$A_3 = \xi_3'^2 - 2\xi_3'\{y_2 \cos \theta_1 + (y_3 - d_1)\sin \theta_1 + 2mH \sin \theta_1\} +$$
$$y_2^2 + (y_3 - d_1)^2 + 4(y_3 - d_1)\sin \theta_1 + 4m^2 H^2$$
$$A_4 = \xi_3'^2 - 2\xi_3'\{y_2 \cos \theta_1 - (y_3 + d_1)\sin \theta_1 + 2mH \sin \theta_1\} +$$
$$y_2^2 + (y_3 - d_1)^2 - 4(y_3 - d_1)mH + 4m^2 H^2$$
$$A_5 = \xi_3'^2 - 2\xi_3'\{y_2 \cos \theta_1 + (y_3 - d_1)\sin \theta_1 - 2mH \sin \theta_1\} +$$
$$y_2^2 + (y_3 - d_1)^2 - 4(y_3 - d_1)mH + 4m^2 H^2$$
$$A_6 = \xi_3'^2 - 2\xi_3'\{y_2 \cos \theta_1 - (y_3 + d_1)\sin \theta_1 - 2mH \sin \theta_1\} +$$
$$y_2^2 + (y_3 + d_1)^2 + 4(y_3 + d_1)mH + 4m^2 H^2$$

Taking inverse Laplace transform we get,

$$(u_1)_2 = \frac{U_1}{2\pi}\psi_1(y_2, y_3, t) \qquad (A13)$$

where

$$\psi_1(y_2, y_3, t) = \int_0^{l_1} f_1(\xi_3') \Big[\frac{y_2 \sin \theta_1 - (y_3 - d_1)\cos \theta_1}{A_1} +$$
$$\frac{y_2 \sin \theta_1 + (y_3 + d_1)\cos \theta_1}{A_2} \Big] d\xi_3' +$$
$$\sum_{m=1}^{\infty} \Big(\frac{\alpha}{\beta} \Big)^m A_m(t) \int_0^{l_1} f_1(\xi_3') \Big\{ \frac{y_2 \sin \theta_1 - 2mH\cos \theta_1 - (y_3 - d_1)\cos \theta_1}{A_3} +$$
$$\frac{y_2 \sin \theta_1 - 2mH\cos \theta_1 + (y_3 + d_1)\cos \theta_1}{A_4} +$$
$$\frac{y_2 \sin \theta_1 + 2mH\cos \theta_1 - (y_3 - d_1)\cos \theta_1}{A_5} +$$
$$\frac{y_2 \sin \theta_1 + 2mH\cos \theta_1 + (y_3 + d_1)\cos \theta_1}{A_6} \Big\} d\xi_3' \qquad (A14)$$

From (15) we get,

$$(\tau_{12})_2 = \frac{\mu_1 U_1}{2\pi} \frac{\partial}{\partial y_2}\psi_1(y_2, y_3, t)$$

21

$$= \frac{\mu_1 U_1}{2\pi}\psi_2(y_2, y_3, t) \qquad (A15)$$

where

$$\psi_2(y_2, y_3, t) = \int_0^{l_1} f_1(\xi_3')\Big[\frac{\xi_3'^2 \sin\theta_1 + \{(y_3-d_1)^2 - y_2^2\}\sin\theta_1 + 2y_2(y_3-d_1)\cos\theta_1 - 2\xi_3'(y_3-d_1)}{A_1^2} +$$

$$\frac{\xi_3'^2 \sin\theta_1 + \{(y_3+d_1)^2 - y_2^2\}\sin\theta_1 - 2y_2(y_3+d_1)\cos\theta_1 + 2\xi_3'(y_3+d_1)}{A_2^2}\Big]d\xi_3' +$$

$$\sum_{m=1}^{\infty}\left(\frac{\alpha}{\beta}\right)^m A_m(t)\int_0^{l_1} f_1(\xi_3')\Big[\frac{1}{A_3^2}\{\xi_3'^2 \sin\theta_1 + \{(y_3 - d_1 + 2mH)^2 - y_2^2\}\times$$

$$\sin\theta_1 + 2y_2(y_3 - d_1 + 2mH)\cos\theta_1 -$$

$$2\xi_3'(y_3 - d_1 + 2mH)\} +$$

$$\frac{1}{A_4^2}\{\xi_3'^2 \sin\theta_1 + \{(y_3 + d_1 - 2mH)^2 - y_2^2\}\times$$

$$\sin\theta_1 - 2y_2(y_3 + d_1 - 2mH)\cos\theta_1 +$$

$$2\xi_3'(y_3 + d_1 - 2mH)\} +$$

$$\frac{1}{A_5^2}\{\xi_3'^2 \sin\theta_1 + \{(y_3 - d_1 - 2mH)^2 - y_2^2\}\times$$

$$\sin\theta_1 + 2y_2(y_3 - d_1 - 2mH)\cos\theta_1 -$$

$$2\xi_3'(y_3 - d_1 - 2mH)\} +$$

$$\frac{1}{A_6^2}\{\xi_3'^2 \sin\theta_1 + \{(y_3 + d_1 + 2mH)^2 - y_2^2\}\times$$

$$\sin\theta_1 - 2y_2(y_3 + d_1 + 2mH)\cos\theta_1 +$$

$$2\xi_3'(y_3 + d_1 + 2mH)\}\Big]d\xi_3' \qquad (A16)$$

and

$$(\tau_{13})_2 = \mu_1\frac{\partial}{\partial y_3}(u_1)_2$$

$$= \frac{\mu_1 U_1}{2\pi}\psi_3(y_2, y_3, t) \qquad (A17)$$

where

$$\psi_3(y_2, y_3, t) = \int_0^{l_1} f_1(\xi_3') \left[\frac{-\xi_3'^2 \cos\,\theta_1 + \{(y_3-d_1)^2 - y_2^2\}\cos\,\theta_1 - 2y_2(y_3-d_1)\sin\,\theta_1 + 2\xi_3' y_2}{A_1^2} + \right.$$

$$\frac{\xi_3'^2 \cos\,\theta_1 + \{y_2^2 - (y_3+d_1)^2\}\cos\,\theta_1 - 2y_2(y_3+d_1)\sin\,\theta_1 - 2\xi_3' y_2}{A_2^2} \left] d\xi_3' + \right.$$

$$\sum_{m=1}^{\infty} \left(\tfrac{\alpha}{\beta}\right)^m A_m(t) \int_0^{l_1} f_1(\xi_3') \left[\tfrac{1}{A_3^2}\{-\xi_3'^2 \cos\,\theta_1 + 2\xi_3' y_2 - \right.$$

$$2y_2(y_3-d_1)\sin\,\theta_1 -$$
$$\{y_2^2 - (y_3-d_1)^2\}\cos\,\theta_1 -$$
$$4mH\{y_2\sin\,\theta_1 - (y_3-d_1)\cos\,\theta_1\} +$$
$$4m^2H^2\cos\,\theta_1\} + \tfrac{1}{A_4^2}\{\xi_3'^2\cos\,\theta_1 - 2\xi_3' y_2 -$$
$$2y_2(y_3+d_1)\sin\,\theta_1 +$$
$$\{y_2^2 - (y_3+d_1)^2\}\cos\,\theta_1 +$$
$$4mH\{y_2\sin\,\theta_1 + (y_3+d_1)\cos\,\theta_1\} -$$
$$4m^2H^2\cos\,\theta_1\} + \tfrac{1}{A_5^2}\{-\xi_3'^2\cos\,\theta_1 + 2\xi_3' y_2 -$$
$$2y_2(y_3-d_1)\sin\,\theta_1 -$$
$$\{y_2^2 - (y_3-d_1)^2\}\cos\,\theta_1 +$$
$$4mH\{y_2\sin\,\theta_1 - (y_3-d_1)\cos\,\theta_1\} +$$
$$4m^2H^2\cos\,\theta_1\} + \theta_1\} + \tfrac{1}{A_6^2}\{\xi_3'^2\cos\,\theta_1 -$$
$$2\xi_3' y_2 - 2y_2(y_3+d_1)\sin\,\theta_1 +$$
$$\{y_2^2 - (y_3+d_1)^2\}\cos\,\theta_1 -$$
$$4mH\{y_2\sin\,\theta_1 + (y_3+d_1)\cos\,\theta_1\} -$$
$$4m^2H^2\cos\,\theta_1\}]d\xi_3' \qquad (A18)$$

where

$$s = \tfrac{\mu_2}{\mu_1}, \ \alpha = \tfrac{\mu_1}{\mu_2} - 1, \ \beta = \tfrac{\mu_1}{\mu_2} + 1, \ a_1 = \frac{\mu_1\mu_2}{\eta(\mu_1 + \mu_2)}, \ b_1 = \frac{2\mu_1\mu_2^2}{\eta(\mu_1^2 - \mu_2^2)}$$

$$e_n(z) = 1 + \tfrac{z}{1!} + \tfrac{z^2}{2!} + \quad + \tfrac{z^n}{n!}, \ e_0(z) = 1,$$

$$A_m(t) = 1 + \sum_{r=1}^{m} \binom{m}{r}\left(\tfrac{2s}{1-s}\right)^r \{1 - e^{-a_1 t}e_n(a_1 t)\}$$

$$B_{rm} = \binom{m}{r}b_1^r, \ A_{rm} = \binom{m}{r}\left(\tfrac{b_1}{a_1}\right)^r$$

From the above solutions we can compute the strain

$$e_{12} = \frac{\partial u_1}{\partial y_2} \qquad (A19)$$

In case of half space

$$G_{13}'(P, Q_1) = \mu_1 \frac{\partial}{\partial x_3} G_1'(P, Q_1)$$
$$G_{12}'(P, Q_1) = \mu_1 \frac{\partial}{\partial x_2} G_1'(P, Q_1)$$

23

where

$$G_1'(P,Q_1) = -\frac{1}{\pi(\mu_1+\bar{\mu}_2)}[\log\sqrt{(x_2-y_2)^2+(x_3-y_3)^2}+$$
$$\log\sqrt{(x_2-y_2)^2+(x_3+y_3)^2}+$$
$$\sum_{m=1}^{\infty}(\frac{\mu_1-\bar{\mu}_2}{\mu_1+\bar{\mu}_2})^m\{\log\sqrt{(x_2-y_2)^2+(x_3+2mH-y_3)^2}+$$
$$\log\sqrt{(x_2-y_2)^2+(x_3+2mH+y_3)^2}\}] \qquad (A20)$$

$$(\bar{u}_1')_2(Q_1) = \frac{U_1}{p}\frac{\mu_1}{\pi(\mu_1+\mu_2)}\int f_1(\xi_3')[\{\frac{y_2\sin\theta_1-(y_3-d_1)\cos\theta_1}{A_1}+\frac{y_2\sin\theta_1+(y_3+d_1)\cos\theta_1}{A_2}\}+$$
$$\sum_{m=1}^{\infty}(\frac{\mu_1-\bar{\mu}_2}{\mu_1+\bar{\mu}_2})^m\{\frac{y_2\sin\theta_1-(y_3-d_1)\cos\theta_1+2mH\cos\theta_1}{A_5}+$$
$$\frac{y_2\sin\theta_1+(y_3+d_1)\cos\theta_1+2mH\cos\theta_1}{A_6}\}]d\xi_3'$$

Therefore taking inverse Laplace transform we get,

$$(u_1')_2(Q_1) = \frac{U_1}{\pi}(1-\frac{s}{1+s}e^{-a_1t})\int_0^{l_1}f_1(\xi_3')[\frac{y_2\sin\theta_1-(y_3-d_1)\cos\theta_1}{A_1}+\frac{y_2\sin\theta_1+(y_3+d_1)\cos\theta_1}{A_2}]d\xi_3'+$$
$$\frac{U_1}{\pi}\sum_{m=1}^{\infty}(\frac{\alpha}{\beta})^m[1+\sum_{r=1}^{m}\binom{m}{r}(\frac{b_1}{a_1})^r\{1-e^{-a_1t}e_{r-1}(a_1t)\}-\frac{s}{1+s}e^{-a_1t}-$$
$$\frac{s}{1+s}\sum_{r=1}^{m}\binom{m}{r}\frac{b_1^r t^r}{\Gamma(r+1)}e^{-a_1t}]\int_0^{l_1}f_1(\xi_3')\{\frac{y_2\sin\theta_1-(y_3-d_1)\cos\theta_1+2mH\cos\theta_1}{A_5}+$$
$$\frac{y_2\sin\theta_1+(y_3+d_1)\cos\theta_1+2mH\cos\theta_1}{A_6}\}]d\xi_3'$$
$$= \frac{U_1}{\pi}\phi_1(y_2,y_3,t) \qquad (A21)$$

where

$$\phi_1(y_2,y_3,t) = (1-\frac{s}{1+s}e^{-a_1t})\int_0^{l_1}f_1(\xi_3')[\frac{y_2\sin\theta_1-(y_3-d_1)\cos\theta_1}{A_1}+$$
$$\frac{y_2\sin\theta_1+(y_3+d_1)\cos\theta_1}{A_2}]d\xi_3'+$$
$$\sum_{m=1}^{\infty}(\frac{1-s}{1+s})^m[1+\sum_{r=1}^{m}\binom{m}{r}(\frac{b_1}{a_1})^r\{1-e^{-a_1t}e_{r-1}(a_1t)\}-\frac{s}{1+s}e^{-a_1t}- \qquad (A22)$$
$$\frac{s}{1+s}\sum_{r=1}^{m}\binom{m}{r}\frac{b_1^r t^r}{\Gamma(r+1)}e^{-a_1t}]\int_0^{l_1}f_1(\xi_3')\{\frac{y_2\sin\theta_1-(y_3-d_1)\cos\theta_1+2mH\cos\theta_1}{A_5}+$$
$$\frac{y_2\sin\theta_1+(y_3+d_1)\cos\theta_1+2mH\cos\theta_1}{A_6}\}]d\xi_3'$$

24

From (A3) we get

$$(\bar{\tau}'_{12})_2 = \bar{\mu}_2 \frac{\partial}{\partial y_2}(\bar{u}'_1)_2$$

$$= \frac{\bar{\mu}_2 U_1}{p}\frac{\mu_1}{\pi(\mu_1+\bar{\mu}_2)}\int_0^{l_1} f_1(\xi'_3)[\frac{\xi'^2_3\sin\,\theta_1+\{(y_3-d_1)^2-y_2^2\}\sin\,\theta_1+2y_2(y_3-d_1)\cos\,\theta_1-2\xi'_3(y_3-d_1)}{A_1^2}+$$

$$\frac{\xi'^2_3\sin\,\theta_1+\{(y_3+d_1)^2-y_2^2\}\sin\,\theta_1-2y_2(y_3+d_1)\cos\,\theta_1+2\xi'_3(y_3+d_1)}{A_2^2}]d\xi'_3+$$

$$\frac{\bar{\mu}_2 U_1}{p}\frac{\mu_1}{\pi(\mu_1+\bar{\mu}_2)}\sum_{m=1}^{\infty}(\frac{\mu_1-\bar{\mu}_2}{\mu_1+\bar{\mu}_2})^m\int_0^{l_1} f_1(\xi'_3)[\frac{1}{A_5^2}\{\xi'^2_3\sin\,\theta_1+\{(y_3-d_1-2mH)^2-y_2^2\}\times$$

$$\sin\,\theta_1+2y_2(y_3-d_1-2mH)\cos\,\theta_1-$$

$$2\xi'_3(y_3-d_1-2mH)\}+$$

$$\frac{1}{A_6^2}\{\xi'^2_3\sin\,\theta_1+\{(y_3+d_1+2mH)^2-y_2^2\}\times$$

$$\sin\,\theta_1-2y_2(y_3+d_1+2mH)\cos\,\theta_1+$$

$$2\xi'_3(y_3+d_1+2mH)\}]d\xi'_3$$

Taking inverse Laplace tranform,

$$(\tau'_{12})_2 = \frac{U_1}{\pi}\phi_2(y_2,y_3,t) \qquad (A23)$$

where

$$\phi_2(y_2,y_3,t) = \frac{\mu_2}{1+s}e^{-a_1 t}\int_0^{l_1} f_1(\xi'_3)[\frac{\xi'^2_3\sin\,\theta_1+\{(y_3-d_1)^2-y_2^2\}\sin\,\theta_1+2y_2(y_3-d_1)\cos\,\theta_1-2\xi'_3(y_3-d_1)}{A_1^2}+$$

$$\frac{\xi'^2_3\sin\,\theta_1+\{(y_3+d_1)^2-y_2^2\}\sin\,\theta_1-2y_2(y_3+d_1)\cos\,\theta_1+2\xi'_3(y_3+d_1)}{A_2^2}]d\xi'_3$$

$$+\frac{\mu_2}{1+s}\sum_{m=1}^{\infty}(\frac{1-s}{1+s})^m e^{-a_1 t}(1+\sum_{r=1}^{m}\frac{B_{rm}t^r}{r!})\times$$

$$\int_0^{l_1} f_1(\xi'_3)[\frac{1}{A_5^2}\{\xi'^2_3\sin\,\theta_1+\{(y_3-d_1-2mH)^2-y_2^2\}\sin\,\theta_1+$$

$$2y_2(y_3-d_1-2mH)\cos\,\theta_1-2\xi'_3(y_3-d_1-2mH)\}+$$

$$\frac{1}{A_6^2}\{\xi'^2_3\sin\,\theta_1+\{(y_3+d_1+2mH)^2-y_2^2\}\sin\,\theta_1-$$

$$2y_2(y_3+d_1+2mH)\cos\,\theta_1+2\xi'_3(y_3+d_1+2mH)\}]d\xi'_3 \qquad\qquad (A24)$$

Similarly,

$$(\tau'_{13})_2 = \frac{U_1}{\pi}\phi_3(y_2,y_3,t) \qquad (A25)$$

25

where,

$$\phi_3(y_2, y_3, t) = \frac{\mu_2}{1+s} e^{-a_1 t} \int\limits_0^{l_1} f_1(\xi_3') \left[\frac{-\xi_3'^2 \cos\,\theta_1 + \{(y_3-d_1)^2 - y_2^2\}\cos\,\theta_1 - 2y_2(y_3-d_1)\sin\,\theta_1 + 2\xi_3' y_2}{A_1^2} + \right.$$

$$\left. \frac{\xi_3'^2 \cos\,\theta_1 - \{(y_3+d_1)^2 - y_2^2\}\cos\,\theta_1 - 2y_2(y_3+d_1)\sin\,\theta_1 - 2\xi_3' y_2}{A_2^2} \right] d\xi_3'$$

$$+ \frac{\mu_2}{1+s} \sum_{m=1}^{\infty} \left(\frac{1-s}{1+s}\right)^m e^{-a_1 t} \left(1 + \sum_{r=1}^{m} \frac{B_{rm} t^r}{r!}\right) \times$$

$$\int\limits_0^{l_1} f_1(\xi_3') \left[\frac{1}{A_5^2} \{ -\xi_3'^2 \cos\,\theta_1 + \{(y_3-d_1)^2 - y_2^2\}\cos\,\theta_1 - \right.$$

$$2y_2(y_3-d_1)\sin\,\theta_1 + 2\xi_3' y_2 +$$

$$4mH\{y_2 \sin\,\theta_1 - (y_3-d_1)\cos\,\theta_1\} + 4m^2 H^2 \cos\,\theta_1\} +$$

$$\frac{1}{A_6^2} \{\xi_3'^2 \cos\,\theta_1 + \{(y_3+d_1)^2 - y_2^2\}\cos\,\theta_1 -$$

$$2y_2(y_3+d_1)\sin\,\theta_1 - 2\xi_3' y_2 -$$

$$\left. 4mH\{y_2 \sin\,\theta_1 + (y_3+d_1)\cos\,\theta_1\} - 4m^2 H^2 \cos\,\theta_1\} \right] d\xi_3' \qquad (A26)$$

Due to movement of fault part F_2 the displacement and stress components are $(u_1)_3$, $(u_1')_3$, $(\tau_{12})_3$, $(\tau_{13})_3$, $(\tau_{12'})_3$, $(\tau_{13'})_3$. Applying similar method taking Green's function

$$G_1(P, Q) = -\frac{1}{\pi(\mu_1 + \bar{\mu}_2)} \left[\log\sqrt{(x_2-y_2)^2 + (x_3-y_3)^2} + \right.$$

$$\log\sqrt{(x_2-y_2)^2 + (x_3+y_3)^2} +$$

$$\sum_{m=1}^{\infty} \left(\frac{\mu_1 - \bar{\mu}_2}{\mu_1 + \bar{\mu}_2}\right)^m \{ \log\sqrt{(x_2-y_2)^2 + (x_3+2mH-y_3)^2} +$$

$$\left. \log\sqrt{(x_2-y_2)^2 + (x_3+2mH+y_3)^2} \} \right] \qquad (A27)$$

For layered medium $(y_3 \leq H)$

and

$$G_1'(P, Q_1) = -\frac{1}{2\pi} \left[\frac{1}{\bar{\mu}_2} \log\sqrt{(x_2-y_2)^2 + (x_3-y_3)^2} + \right.$$

$$\frac{1}{\bar{\mu}_2} \frac{\bar{\mu}_2 - \mu_1}{\bar{\mu}_2 + \mu_1} \log\sqrt{(x_2-y_2)^2 + (x_3-2H+y_3)^2} +$$

$$\frac{4\mu_1}{(\mu_1+\bar{\mu}_2)^2} \log\sqrt{(x_2-y_2)^2 + (x_3+y_3)^2} +$$

$$\left. \frac{4\mu_1}{(\mu_1+\bar{\mu}_2)^2} \sum_{m=1}^{\infty} \left(\frac{\mu_1 - \bar{\mu}_2}{\mu_1 + \bar{\mu}_2}\right)^m \log\sqrt{(x_2-y_2)^2 + (x_3+2mH+y_3)^2} \right] \qquad (A28)$$

For viscoelastic half space $(y_3 \geq H)$

26

We get,

$$\left.\begin{array}{l}(u_1)_3(Q) = \frac{U_2}{\pi}\psi'_1(y_2,y_3,t) \\[2mm] (\tau_{12})_3 = \frac{\mu_1 U_2}{\pi}\psi'_2(y_2,y_3,t) \\[2mm] (\tau_{13})_3 = \frac{\mu_1 U_2}{\pi}\psi'_3(y_2,y_3,t)\end{array}\right\} \qquad (A29)$$

where

$$\left.\begin{array}{l}\psi'_1(y_2,y_3,t) = \frac{s}{1+s}e^{-a_1 t}\int\limits_0^{l_2} f_2(\xi''_3)\big[\frac{(y_2-l_1\cos\ \theta_1)\ \sin\ \theta_2-(y_3-H)\ \cos\ \theta_2}{B_1}+ \\[3mm] \qquad\qquad \frac{(y_2-l_1\cos\ \theta_1)\ \sin\ \theta_2+(y_3+H)\ \cos\ \theta_2}{B_2}\big]d\xi''_3+ \\[3mm] \frac{s}{1+s}e^{-a_1 t}\sum\limits_{m=1}^{\infty}\big(\frac{1-s}{1+s}\big)^m e^{-a_1 t}\big(1+\sum\limits_{r=1}^{m}\frac{B_{rm}t^r}{r!}\big)\times \\[3mm] \int\limits_0^{l_2} f_2(\xi''_3)\big[\frac{(y_2-l_1\cos\ \theta_1)\ \sin\ \theta_2-(y_3-H)\ \cos\ \theta_2+2mH\cos\ \theta_2}{B_5}+ \\[3mm] \qquad\qquad \frac{(y_2-l_1\cos\ \theta_1)\ \sin\ \theta_2+(y_3+H)\ \cos\ \theta_2+2mH\cos\ \theta_2}{B_6}\big]d\xi''_3 \end{array}\right\} \qquad (A30)$$

$$\left.\begin{array}{l}\psi'_2(y_2,y_3,t) = \frac{s}{1+s}e^{-a_1 t}\int\limits_0^{l_2} f_2(\xi''_3)\big[\frac{\sin\ \theta_2}{B_1}+\frac{\sin\ \theta_2}{B_2}+ \\[3mm] \qquad \frac{2\{(y_2-l_1\cos\ \theta_1)\ \sin\ \theta_2-(y_3-H)\ \cos\ \theta_2\}(l_1\cos\ \theta_1+\xi''_3\cos\ \theta_2-y_2)}{B_1^2}+ \\[3mm] \qquad \frac{2\{(y_2-l_1\cos\ \theta_1)\ \sin\ \theta_2-(y_3+H)\ \cos\ \theta_2\}(l_1\cos\ \theta_1+\xi''_3\cos\ \theta_2-y_2)}{B_2^2}\big]d\xi''_3 \\[3mm] +\frac{s}{1+s}e^{-a_1 t}\sum\limits_{m=1}^{\infty}\big(\frac{1-s}{1+s}\big)^m\big(1+\sum\limits_{r=1}^{m}\frac{B_{rm}t^r}{r!}\big)\times \\[3mm] \int\limits_0^{l_2} f_2(\xi''_3)\big[\frac{\sin\ \theta_2}{B_5}+\frac{\sin\ \theta_6}{B_6}+ \\[3mm] \qquad \frac{2\{(y_2-l_1\cos\ \theta_1)\ \sin\ \theta_2-(y_3-H)\ \cos\ \theta_2+2mH\cos\ \theta_2\}(l_1\cos\ \theta_1+\xi''_3\cos\ \theta_2-y_2)}{B_5^2}+ \\[3mm] \qquad \frac{2\{(y_2-l_1\cos\ \theta_1)\ \sin\ \theta_2+(y_3+H)\ \cos\ \theta_2+2mH\cos\ \theta_2\}(l_1\cos\ \theta_1+\xi''_3\cos\ \theta_2-y_2)}{B_6^2}\big]d\xi''_3 \end{array}\right\} \qquad (A31)$$

$$\psi_3'(y_2, y_3, t) = \frac{s}{1+s} e^{-a_1 t} \int_0^{l_2} f_2(\xi_3'') [-\frac{\cos\,\theta_2}{B_1} + \frac{\cos\,\theta_2}{B_2} +$$

$$\frac{2\{(y_2 - l_1\cos\,\theta_1)\,\sin\,\theta_2 - (y_3 - H)\,\cos\,\theta_2\}(H + \xi_3''\sin\,\theta_2 - y_3)}{B_1^2} -$$

$$\frac{2\{(y_2 - l_1\cos\,\theta_1)\,\sin\,\theta_2 + (y_3 + H)\,\cos\,\theta_2\}(H + \xi_3''\sin\,\theta_2 + y_3)}{B_2^2}] d\xi_3''$$

$$+\frac{s}{1+s} e^{-a_1 t} \sum_{m=1}^{\infty} (\frac{1-s}{1+s})^m (1 + \sum_{r=1}^{m} \frac{B_{rm}t^r}{r!}) \times$$

$$\int_0^{l_2} f_2(\xi_3'') [-\frac{\cos\,\theta_2}{B_5} + \frac{\cos\,\theta_2}{B_6} +$$

$$\frac{2\{(y_2 - l_1\cos\,\theta_1)\,\sin\,\theta_2 - (y_3 - H)\,\cos\,\theta_2 - 2mH\cos\,\theta_2\}(H + 2mH + \xi_3''\sin\,\theta_2 - y_3)}{B_5^2} -$$

$$\frac{2\{(y_2 - l_1\cos\,\theta_1)\,\sin\,\theta_2 + (y_3 + H)\,\cos\,\theta_2 + 2mH\cos\,\theta_2\}(H + 2mH + \xi_3''\sin\,\theta_2 + y_3)}{B_6^2}] d\xi_3''$$

$$\left. \right\} \qquad (A32)$$

and

$$(u_1')_3(Q_1) = \frac{U_2}{2\pi} \phi_1'(y_2, y_3, t)$$

$$(\tau_{12}')_3 = \frac{U_2}{2\pi} \phi_2'(y_2, y_3, t) \qquad (A33)$$

$$(\tau_{13}')_3 = \frac{U_2}{2\pi} \phi_3'(y_2, y_3, t)$$

where

$$\phi_1'(y_2, y_3, t) = \int_0^{l_2} f_2(\xi_3'') [\frac{(y_2 - l_1\cos\,\theta_1)\sin\,\theta_2 - (y_3 - H)\cos\,\theta_2}{B_1} +$$

$$(\frac{2s}{1+s} e^{-a_1 t} - 1)\frac{(y_2 - l_1\cos\,\theta_1)\sin\,\theta_2 + (y_3 - H)\cos\,\theta_2}{B_7} +$$

$$(1 - \frac{\alpha^2}{\beta^2} A_2(t))\frac{(y_2 - l_1\cos\,\theta_1)\sin\,\theta_2 + (y_3 + H)\cos\,\theta_2}{B_2} +$$

$$\{\sum_{m=1}^{\infty} (\frac{\alpha}{\beta})^m A_m(t) - \sum_{m=1}^{\infty} (\frac{\alpha}{\beta})^{m+2} A_{m+2}(t)\} \times$$

$$\frac{(y_2 - l_1\cos\,\theta_1)\sin\,\theta_2 + (y_3 + H)\cos\,\theta_2 + 2mH\cos\,\theta_2}{B_6}] d\xi_3''$$

$$\left. \right\} \qquad (A34)$$

$$\phi_2'(y_2, y_3, t) = \int_0^{l_2} f_2(\xi_3'') \mu_2 e^{-\frac{\mu_2 t}{\eta}} \left[\frac{\sin \theta_2}{B_1} + \right.$$

$$\left. \frac{2\{(y_2 - l_1 \cos \theta_1)\sin \theta_2 - (y_3 - H)\cos \theta_2\}(l_1 \cos \theta_1 + \xi_3'' \cos \theta_2 - y_2)}{B_1^2} \right] d\xi_3'' +$$

$$\mu_2 \left(e^{-\frac{\mu_2 t}{\eta}} - \frac{2}{1+s} e^{-a_1 t} \right) \int_0^{l_2} f_2(\xi_3'') \left[\frac{\sin \theta_2}{B_7} + \right.$$

$$\left. \frac{2\{(y_2 - l_1 \cos \theta_1)\sin \theta_2 + (y_3 - H)\cos \theta_2\}(l_1 \cos \theta_1 + \xi_3'' \cos \theta_2 - y_2)}{B_7^2} \right] d\xi_3'' +$$

$$\left\{ \frac{4\mu_2}{1+s} e^{-a_1 t} - \mu_1 + \mu_1 \left(\frac{\alpha}{\beta}\right)^2 A_2(t) \right\} \int_0^{l_2} f_2(\xi_3'') \left[\frac{\sin \theta_2}{B_2} + \right.$$

$$\left. \frac{2\{(y_2 - l_1 \cos \theta_1)\sin \theta_2 + (y_3 + H)\cos \theta_2\}(l_1 \cos \theta_1 + \xi_3'' \cos \theta_2 - y_2)}{B_2^2} \right] d\xi_3'' +$$

$$\sum_{m=1}^{\infty} \left\{ \frac{4\mu_2}{1+s} \left(\frac{1-s}{1+s}\right)^m e^{-a_1 t} \left(1 + \sum_{r=1}^{m} \frac{B_{rm} t^r}{r!}\right) - \mu_1 \left(\frac{\alpha}{\beta}\right)^m A_m(t) + \right.$$

$$\left. \mu_1 \left(\frac{\alpha}{\beta}\right)^{m+2} A_{m+2}(t) \right\} \int_0^{l_2} f_2(\xi_3'') \left[\frac{\sin \theta_2}{B_6} + \right.$$

$$\left. \frac{2\{(y_2 - l_1 \cos \theta_1)\sin \theta_2 + (y_3 + H)\cos \theta_2 + 2mH \cos \theta_2\}(l_1 \cos \theta_1 + \xi_3'' \cos \theta_2 - y_2)}{B_6^2} \right] d\xi_3'' \qquad (A35)$$

$$\phi_3'(y_2, y_3, t) = \mu_2 e^{-\frac{\mu_2 t}{\eta}} \int_0^{l_2} f_2(\xi_3'') \left[-\frac{\cos \theta_2}{B_1} + \right.$$

$$\left. \frac{2\{(y_2 - l_1 \cos \theta_1)\sin \theta_2 - (y_3 - H)\cos \theta_2\}(H + \xi_3'' \cos \theta_2 - y_3)}{B_1^2} \right] d\xi_3'' +$$

$$\mu_2 \left(e^{-\frac{\mu_2 t}{\eta}} - \frac{2}{1+s} e^{-a_1 t} \right) \int_0^{l_2} f_2(\xi_3'') \left[\frac{\cos \theta_2}{B_7} - \right.$$

$$\left. \frac{2\{(y_2 - l_1 \cos \theta_1)\sin \theta_2 + (y_3 - H)\cos \theta_2\}(H + \xi_3'' \cos \theta_2 - 2H + y_3)}{B_7^2} \right] d\xi_3'' +$$

$$\left\{ \frac{4\mu_2}{1+s} e^{-a_1 t} - \mu_1 + \mu_1 \left(\frac{\alpha}{\beta}\right)^2 A_2(t) \right\} \int_0^{l_2} f_2(\xi_3'') \left[\frac{\cos \theta_2}{B_2} - \right.$$

$$\left. \frac{2\{(y_2 - l_1 \cos \theta_1)\sin \theta_2 + (y_3 + H)\cos \theta_2\}(H + \xi_3'' \cos \theta_2 + y_3)}{B_2^2} \right] d\xi_3'' +$$

$$\sum_{m=1}^{\infty} \left\{ \frac{4\mu_2}{1+s} \left(\frac{1-s}{1+s}\right)^m e^{-a_1 t} \left(1 + \sum_{r=1}^{m} \frac{B_{rm} t^r}{r!}\right) - \mu_1 \left(\frac{\alpha}{\beta}\right)^m A_m(t) + \right.$$

$$\left. \mu_1 \left(\frac{\alpha}{\beta}\right)^{m+2} A_{m+2}(t) \right\} \int_0^{l_2} f_2(\xi_3'') \left[\frac{\cos \theta_2}{B_6} - \right.$$

$$\left. \frac{2\{(y_2 - l_1 \cos \theta_1)\sin \theta_2 + (y_3 + H)\cos \theta_2 + 2mH \cos \theta_2\}(H + \xi_3'' \cos \theta_2 + 2mH + y_3)}{B_6^2} \right] d\xi_3'' \qquad (A36)$$

where

$$
\left.
\begin{aligned}
B_1 &= \xi_3''^2 - 2\xi_3''\{(y_2 - l_1 cos\theta_1)cos\ \theta_2 + (y_3 - H)sin\ \theta_2\}+ \\
&\quad (y_3 - H)^2 + (y_2 - l_1 cos\ \theta_1)^2 \\
B_2 &= \xi_3''^2 - 2\xi_3''\{(y_2 - l_1 cos\theta_1)cos\ \theta_2 - (y_3 + H)sin\ \theta_2\}+ \\
&\quad (y_3 + H)^2 + (y_2 - l_1 cos\ \theta_1)^2 \\
B_5 &= \xi_3''^2 - 2\xi_3''\{(y_2 - l_1 cos\theta_1)cos\ \theta_2 + (y_3 - H)sin\ \theta_2 - 2mH sin\ \theta_2\}+ \\
&\quad (y_3 - H)^2 + (y_2 - l_1 cos\ \theta_1)^2 + 4m^2 H^2 - 4mH(y_3 - H) \\
B_6 &= \xi_3''^2 - 2\xi_3''\{(y_2 - l_1 cos\theta_1)cos\ \theta_2 - (y_3 + H)sin\ \theta_2 - 2mH sin\ \theta_2\}+ \\
&\quad (y_3 + H)^2 + (y_2 - l_1 cos\ \theta_1)^2 + 4m^2 H^2 + 4mH(y_3 + H) \\
B_7 &= \xi_3''^2 - 2\xi_3''\{(y_2 - l_1 cos\theta_1)cos\ \theta_2 - (y_3 - H)sin\ \theta_2\}+ \\
&\quad (y_3 - H)^2 + (y_2 - l_1 cos\ \theta_1)^2
\end{aligned}
\right\} \quad (A37)
$$