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On Inverse Linear Fractional Programming Problem

Sanjay Jain[1], Nitin Arya[2,3,*]

[1] *Department of Mathematical Sciences, Government College Ajmer, Affiliated to M.D.S. University, Ajmer, India*
[2] *Department of Mathematics, Government Engineering College, Jhalawar, Affiliated to Rajasthan Technical University, Kota, India*
[3] *Bhagwant University, Ajmer, India*

Abstract. In this paper, we have proposed an inverse model for linear fractional programming (LFP) problems, in which the cost coefficients, technical coefficients and right hand side vector are adjusted as little as possible so that the given feasible or infeasible solution becomes optimal. In our proposed method, a nonnegative solution x^0 is taken and have adjusted the model parameters as little as possible (under l_1 or l_2 measure) by taking following cases (i) adjusting cost coefficients (ii) adjusting cost, constraint coefficients and right hand side vector (iii) adjusting cost and constraint coefficients (iv) adjusting cost and right hand side vector. Complementary slackness conditions along with some standard transformations and MATLAB is used for optimal solution. The method has been illustrated by a numerical example also.

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1. Introduction

When solving an optimization problem, we assume that all the parameters associated with the decision variable in the objective function and constraints set are known and our object is to find the optimal solution. Practically it is difficult to determine all model parameters with precision but it is plausible to call a solution x^0 which is not optimal under the present parameters and we wish to adjust some or all model parameters as less as possible so that x^0 becomes an optimal solution.

Burton and Toint [5] were the first who investigate the inverse optimization for shortest path problem under l_2 norm, since then a lot of work has been done on inverse optimization but most of the work is based on combinatorial optimization problems. Zhang and Liu [18]

*Corresponding author.

Email addresses: `drjainsanjay@gmail.com` (S. Jain), `nitin.arya1234@gmail.com` (N. Arya)

have first been calculated some inverse linear programming problem and further investigated inverse linear programming problems in [19]. Ahuja and Orlin [1] provide various references in the area of inverse optimization and compile several applications in network flow problems with unit weight and develop combinatorial proofs of correctness. Huang and Liu [10] and Amin and Emrouznejad [2], have considered applications of inverse problem. Yibing, Tiesong and Zhongping [17] worked on inverse optimal value problem. Zhang and Zhang [20–22] worked on inverse quadratic programming problems, and Wang [16] has given the cutting plane algorithm for inverse integer programming problem. Hladik [9] have first been considered inverse problem for generalized linear fractional programming. They have shown that how much data of a generalized linear fractional program can vary such that the optimal values do not exceed some prescribed bounds.

The linear fractional programming problem seeks to optimize the objective function of non-negative variables of quotient form with linear functions in numerator and denominator subject to a set of linear and homogeneous constraints. Bajanilov [3] compiled the literature of Linear Fractional Programming: Theory, Methods, Applications and Software in the form of book. Chadha [6, 7], Charnes-Cooper [8], Kantiswarup [15], Naganathan and Sakthivel [14], Jain and Mangal [11, 12], Jain, Mangal and Parihar [13], Borza, Rambely, and Saraj [4] and many researchers gave different methods for solving linear fractional programming problem.

In the following section, it has been described in brief how inverse optimization can be applied on LFP problem. In our proposed method, first we have obtained the dual and complementary slackness conditions for LFP. For obtaining the required optimal solution by inverse optimization; we adjust the model parameters as little as possible (under l_1 or l_2 measure). Then applying the complementary slackness conditions along with some standard transformation the inverse LFP reduces to a non linear programming problem having large number of variables. The reduced problem can be solved by the optimization software packages like MATLAB, LINGO etc.

2. Mathematical Formulation of Inverse Linear Fractional Programming Problem

The standard linear fractional programming problem is given as follows:

$$\begin{array}{lll} \text{Maximize} & z = \frac{\Sigma_{j\in J} c_j x_j + c_0}{\Sigma_{j\in J} d_j x_j + d_0} & \\ \text{Subject to ,} & \Sigma_{j\in J} a_{ij} x_j \leq b_i & \text{for all} \quad i \in I \\ & x_j \geq 0 & \text{for all} \quad j \in J, \end{array} \tag{1}$$

where I denote the index set of constraints and J is the index set of decision variables. If it is assumed that $\Sigma_{j\in J} d_j x_j + d_0 > 0$ in the feasible region, objective function is continuously differentiable and the feasible set is regular then the dual of above LFP problem given by

Chadha [7] is the following linear programming problem:

$$\begin{array}{lll} \text{Minimize} & z & \\ \text{Subject to,} & \Sigma_{i\in I} a_{ij} y_i + d_j z - v_j = c_j \quad \text{for all} & j \in J \\ & -\Sigma_{i\in I} b_i y_i + d_0 z = c_0 & \\ & y_i \geq 0, v_j \geq 0 \quad \text{for all} & i \in I, j \in J, \end{array} \tag{2}$$

where y_i is the dual variable associated with constraint in primal problem (1) and v_j is the dual variable associated with each $x_j \geq 0$.

We know by the optimality condition for LFP that the primal solution x and the dual solution (y, v) are optimal for their respective problems if they satisfy the following complementary slackness condition:

$$\begin{array}{llll} \text{If} & \Sigma_{j\in J} a_{ij} x_j < b_i & \text{then} & y_i = 0 \\ \text{If} & x_j > 0 & \text{then} & v_j = 0. \end{array} \tag{3}$$

If we have a feasible or infeasible solution x^0 of the LFP problem then inverse problem is to adjust the model parameters as less as possible so that x^0 becomes an optimal solution of the modified problem. Here we are considering four different cases of adjusting parameters.

2.1. Change in Cost Coefficients

Let us assume a feasible solution x^0 and to transform it to an optimal one, we adjust the cost coefficients c_j to c'_j and d_j to d'_j. If we define B $= \{i : \Sigma_{j\in J} a_{ij} x^0_j = b_i\}$, $J_1 = \{j : x^0_j = 0\}$ and $J_2 = \{j : x^0_j > 0\}$, then the complementary slackness conditions can be restate as:

$$\begin{array}{lll} y_i = 0 & \text{for all} & i \notin B \\ v_j = 0 & \text{for all} & j \in J_2. \end{array} \tag{4}$$

The feasible solution x^0 will be the optimal solution of the primal LFP problem (1), where c_j's and d_j's are replaced with c'_j's and d'_j's, if there exist a dual solution (y, v) that satisfy the constraints of dual problem (2) where c_j replaced by c'_j and d_j replaced by d'_j and both primal dual pair satisfy the complementary slackness conditions. Using complementary slackness conditions in the constraints of (2) provide us the following characteristics of the adjusted cost vectors:

$$\begin{array}{lll} \Sigma_{i\in B} a_{ij} y_i + d'_j z - v_j = c'_j & \text{for all} & j \in J_1 \\ \Sigma_{i\in B} a_{ij} y_i + d'_j z = c'_j & \text{for all} & j \in J_2 \\ -\Sigma_{i\in B} b_i y_i + d_0 z = c_0 & & \\ y_i \geq 0, v_j \geq 0 & \text{for all} & i \in B, j \in J. \end{array} \tag{5}$$

Inverse problem is to minimize $||d' - d||_p$ and $||c' - c||_p$, where $||.||_p$ is some selected l_p norm defined as $||a - b||_p = [\Sigma |a_j - b_j|^p]^{\frac{1}{p}}$. We know that minimizing $[\Sigma |a_j - b_j|^p]^{\frac{1}{p}}$ is equivalent to minimizing $\Sigma|a_j - b_j|^p$, so the inverse problem under l_p norm is as follows:

$$\begin{array}{lll} \text{Minimize} & [\Sigma_{j\in J}|d'_j - d_j|^p + \Sigma_{j\in J}|c'_j - c_j|^p] & \\ \text{Subject to,} & \Sigma_{i\in B}a_{ij}y_i + d'_j z - v_j = c'_j & \text{for all} \quad j \in J_1 \\ & \Sigma_{i\in B}a_{ij}y_i + d'_j z = c'_j & \text{for all} \quad j \in J_2 \\ & -\Sigma_{i\in B}b_i y_i + d_0 z = c_0 & \\ & y_i \geq 0, v_j \geq 0 & \text{for all} \quad i \in B, j \in J. \end{array} \tag{6}$$

Let $c'_j = c_j + \alpha_j - \beta_j$ and $\alpha_j \geq 0$, $\beta_j \geq 0$,for all $j \in J$, where α and β are respective increment and decrement in c. Here $\alpha_j\beta_j = 0$, i.e. α_j and β_j can never be positive at the same time, similarly $d'_j = d_j + \gamma_j - \delta_j$, $\gamma_j \geq 0$, $\delta_j \geq 0$ and $\gamma_j\delta_j = 0$ for all $j \in J$. Using these transformations, minimizing $|c'_j - c_j|$ and $|d'_j - d_j|$ is equivalent to minimize $\alpha_j + \beta_j$ and $\gamma_j + \delta_j$ respectively, therefore the inverse linear fractional programming (ILFP) problem under l_1 norm is as follows:

$$\begin{array}{lll} \text{Minimize} & \Sigma_{j\in J}(\alpha_j + \beta_j + \gamma_j + \delta_j) & \\ \text{Subject to,} & \Sigma_{i\in B}a_{ij}y_i + (d_j + \gamma_j - \delta_j)z - v_j = c_j + \alpha_j - \beta_j & \text{for all} \quad j \in J_1 \\ & \Sigma_{i\in B}a_{ij}y_i + (d_j + \gamma_j - \delta_j)z = c_j + \alpha_j - \beta_j & \text{for all} \quad j \in J_2 \\ & -\Sigma_{i\in B}b_i y_i + d_0 z = c_0 & \\ & y_i, v_j, \alpha_j, \beta_j, \gamma_j, \delta_j \geq 0 & \text{for all} \quad i \in B, j \in J. \end{array} \tag{7}$$

If we also want to modify the constant terms in numerator and denominator, then by replacing c_0 and d_0 with c'_0 and d'_0 and substituting $c'_0 = c_0 + \alpha_0 - \beta_0$ and $d'_0 = d_0 + \gamma_0 - \delta_0$ in equation (7), the inverse linear fractional programming (ILFP) problem under l_1 norm will be as follows:

$$\begin{array}{lll} \text{Minimize} & \Sigma_{j\in J\cup\{0\}}(\alpha_j + \beta_j + \gamma_j + \delta_j) & \\ \text{Subject to,} & \Sigma_{i\in B}a_{ij}y_i + (d_j + \gamma_j - \delta_j)z - v_j = c_j + \alpha_j - \beta_j & \text{for all} \quad j \in J_1 \\ & \Sigma_{i\in B}a_{ij}y_i + (d_j + \gamma_j - \delta_j)z = c_j + \alpha_j - \beta_j & \text{for all} \quad j \in J_2 \\ & -\Sigma_{i\in B}b_i y_i + (d_0 + \gamma_0 - \delta_0)z = c_0 + \alpha_0 - \beta_0 & \\ & y_i \geq 0, v_j \geq 0 & \text{for all} \quad i \in B, j \in J \\ & \alpha_j, \beta_j, \gamma_j, \delta_j \geq 0 & \text{for all} \quad j \in J \cup \{0\}. \end{array} \tag{8}$$

If we consider l_2 norm then the inverse problem will be given as:

$$\begin{array}{lll} \text{Minimize} & \Sigma_{j\in J\cup\{0\}}(\alpha_j^2 + \beta_j^2 + \gamma_j^2 + \delta_j^2) & \\ \text{Subject to,} & \Sigma_{i\in B}a_{ij}y_i + (d_j + \gamma_j - \delta_j)z - v_j = c_j + \alpha_j - \beta_j & \text{for all} \quad j \in J_1 \\ & \Sigma_{i\in B}a_{ij}y_i + (d_j + \gamma_j - \delta_j)z = c_j + \alpha_j - \beta_j & \text{for all} \quad j \in J_2 \\ & -\Sigma_{i\in B}b_i y_i + (d_0 + \gamma_0 - \delta_0)z = c_0 + \alpha_0 - \beta_0 & \\ & y_i \geq 0, v_j \geq 0 & \text{for all} \quad i \in B, j \in J \\ & \alpha_j, \beta_j, \gamma_j, \delta_j \geq 0 & \text{for all} \quad j \in J \cup \{0\}. \end{array} \tag{9}$$

2.2. Change in Cost, Constraint Coefficients and Right Hand Side Vector

Let us assume that x^0 is an infeasible solution. In order to remove the infeasibility, we have to adjust the parameters associated with x^0 in the constraint set of primal problem (1).

Let a'_{ij} and b'_i are the respective adjusted values of a_{ij} and b_i then by equation (1), we have

$$\Sigma_{j\in J} a'_{ij} x^0_j \le b'_i. \tag{10}$$

If c'_j and d'_j are respective adjusted values of c_j and d_j (as defined in 2.1), then the constraint set of (6) with a_{ij} replaced by a'_{ij} along with (10) give us the following characteristics of adjusted parameters:

$$\begin{array}{lll} \Sigma_{i\in I} a'_{ij} y_i + d'_j z - v_j = c'_j & \text{for all} & j \in J_1 \\ \Sigma_{i\in I} a'_{ij} y_i + d'_j z = c'_j & \text{for all} & j \in J_2 \\ -\Sigma_{i\in I} b'_i y_i + d'_0 z = c'_0 & & \\ \Sigma_{j\in J} a'_{ij} x^0_j \le b'_i & \text{for all} & i \in I \\ y_i \ge 0, v_j \ge 0 & \text{for all} & i \in I, j \in J, \end{array} \tag{11}$$

where J_1 and J_2 are index sets as defined in previous section.

Let us define

$$\begin{array}{llllll} & a'_{ij} = a_{ij} + \eta_{ij} - \xi_{ij}, \eta_{ij} \ge 0, \xi_{ij} \ge 0 & \text{and} & \eta_{ij}\xi_{ij} = 0 & \text{for all} & i \in I, j \in J \\ \text{and} & b'_i = b_i + p_i - q_i, p_i \ge 0, q_i \ge 0 & \text{and} & p_i q_i = 0 & \text{for all} & i \in I. \end{array} \tag{12}$$

Now the inverse problem is to find the adjusted values of parameters which are differ from the original values of parameters as little as possible and make the given solution optimal, therefore substituting the values of adjusted parameters in (11) and using l_2 norm same as in previous case, the inverse LFP problem can be formulated as:

$$\text{Minimize} \quad \Sigma_{j\in J\cup\{0\}}(\alpha^2_j + \beta^2_j + \gamma^2_j + \delta^2_j) + \Sigma_{i\in I}(p^2_i + q^2_i) + \Sigma_{i\in I}\Sigma_{j\in J}(\eta^2_{ij} + \xi^2_{ij})$$

$$\begin{array}{llll} \text{Subject to,} & \Sigma_{i\in I}(a_{ij} + \eta_{ij} - \xi_{ij})y_i + (d_j + \gamma_j - \delta_j)z - v_j = c_j + \alpha_j - \beta_j & \text{for all} & j \in J_1 \\ & \Sigma_{i\in I}(a_{ij} + \eta_{ij} - \xi_{ij})y_i + (d_j + \gamma_j - \delta_j)z = c_j + \alpha_j - \beta_j & \text{for all} & j \in J_2 \\ & -\Sigma_{i\in I}(b_i + p_i - q_i)y_i + (d_0 + \gamma_0 - \delta_0)z = c_0 + \alpha_0 - \beta_0 & & \\ & \Sigma_{j\in J}(a_{ij} + \eta_{ij} - \xi_{ij})x^0_j \le b_i + p_i - q_i & \text{for all} & i \in I \\ & y_i, v_j, p_i, q_i, \eta_{ij}, \xi_{ij} \ge 0 & \text{for all} & i \in I, j \in J \\ & \alpha_j, \beta_j, \gamma_j, \delta_j \ge 0 & \text{for all} & j \in J \cup \{0\}. \end{array} \tag{13}$$

2.3. Change in Cost Coefficients and Constraint Coefficients

If we want to adjust only the coefficients in the constraint set and objective function then by substituting $p_i = q_i = 0$ in the equation (13), we get the inverse LFP problem.

2.4. Change in Cost Coefficients and Right Hand Side Vector

If we want to adjust only the coefficients in objective function and right hand side vector then by substituting $\eta_{ij} = \xi_{ij} = 0$ in the equation (13), we get the inverse LFP problem.

3. Numerical Example

Let us consider a primal LFP problem

$$\begin{aligned} \text{Maximize } z &= \tfrac{2x_1+3x_2+1}{x_1+x_2+4} \\ \text{Subject to ,} \quad & x_1+x_2 \le 4 \\ & 3x_1+x_2 \le 6 \\ & x_1, x_2 \ge 0. \end{aligned} \tag{14}$$

The optimal solution of this LFP is $x_1 = 0, x_2 = 4$ with the objective value $z = 1.625$. Let us assume a feasible solution $x_1^0 = 1, x_2^0 = 3$, and we want to make it an optimal solution using inverse optimization. Here we can see that both the constraints are binding with respect to x^0 and also $x_1^0 > 0, x_2^0 > 0$, therefore by complementary slackness condition for LFP, the dual constraints correspond to x_1 and x_2 will be binding i.e. $v_1 = 0, v_2 = 0$ and the dual variable will be non negative. If we adjust only cost coefficients, then the inverse LFP problem under l_2 norm is as follows:

$$\begin{aligned} \text{Minimize} \quad & \Sigma_{j=0}^{2}(\alpha_j^2+\beta_j^2+\gamma_j^2+\delta_j^2) \\ \text{Subject to,} \quad & \Sigma_{i=1}^{2} a_{ij} y_i + (d_j+\gamma_j-\delta_j)z = c_j+\alpha_j-\beta_j, j = 1,2 \\ & -\Sigma_{i=1}^{2} b_i y_i + (d_0+\gamma_0-\delta_0)z = c_0+\alpha_0-\beta_0 \\ & y_i \ge 0, i = 1,2 \text{ and } \alpha_j, \beta_j, \gamma_j, \delta_j \ge 0, j = 0,1,2. \end{aligned} \tag{15}$$

Substituting the values and in the above equation and simplifying, we get the following non linear programming problem:

$$\begin{aligned} \text{Minimize} \quad & (\alpha_0^2+\beta_0^2+\alpha_1^2+\beta_1^2+\alpha_2^2+\beta_2^2+\gamma_0^2+\delta_0^2+\gamma_1^2+\delta_1^2+\gamma_2^2+\delta_2^2) \\ \text{Subject to,} \quad & y_1+3y_2+(1+\gamma_1-\delta_1)z-\alpha_1+\beta_1 = 2 \\ & y_1+y_2+(1+\gamma_2-\delta_2)z-\alpha_2+\beta_2 = 3 \\ & -4y_1-6y_2+(4+\gamma_0-\delta_0)z-\alpha_0+\beta_0 = 1 \\ & y_1, y_2, \alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1, \beta_2\gamma_0, \gamma_1, \gamma_2, \delta_0, \delta_1, \delta_2 \ge 0. \end{aligned} \tag{16}$$

Optimal solution of the inverse problem using MATLAB is

$$\begin{aligned} & \alpha_1 = 0.1876, \delta_1 = 0.2643, \beta_2 = 0.1474, \gamma_2 = 0.2077 \\ & \delta_0 = 0.0141, \alpha_0 = 0.01, y_1 = 1.1512, z = 1.4087. \end{aligned} \tag{17}$$

Using the values from equation (17) in the objective function of (14), we obtain the following modified objective function of the LFP problem:

$$\text{Maximize } z = \tfrac{2.1876x_1+2.8526x_2+1.01}{0.7357x_1+1.2077x_2+3.9859}. \tag{18}$$

If we consider an infeasible solution $x_1 = 2, x_2 = 1$ and adjust all the parameters together then the inverse problem under l_2 norm is as follows:

$$\begin{array}{ll} \text{Minimize} & (\alpha_0^2+\beta_0^2+\alpha_1^2+\beta_1^2+\alpha_2^2+\beta_2^2+\gamma_0^2+\delta_0^2+\gamma_1^2+\delta_1^2+\gamma_2^2+\delta_2^2 \\ & +\eta_{11}^2+\xi_{11}^2+\eta_{12}^2+\xi_{12}^2+\eta_{21}^2+\xi_{21}^2+\eta_{22}^2+\xi_{22}^2+p_1^2+q_1^2+p_2^2+q_2^2) \\ \text{Subject to,} & (1+\eta_{11}-\xi_{11})y_1+(3+\eta_{21}-\xi_{21})y_2+(1+\gamma_1-\delta_1)z-\alpha_1+\beta_1=2 \\ & (1+\eta_{12}-\xi_{12})y_1+(1+\eta_{22}-\xi_{22})y_2+(1+\gamma_2-\delta_2)z-\alpha_2+\beta_2=3 \\ & -(4+p_1-q_1)y_1-(6+p_2-q_2)y_2+(4+\gamma_0-\delta_0)z-\alpha_0+\beta_0=1 \\ & 2\eta_{11}-2\xi_{11}+\eta_{12}-\xi_{12}-p_1+q_1=1 \\ & 2\eta_{21}-2\xi_{21}+\eta_{22}-\xi_{22}-p_2+q_2=-1 \\ & y_1,y_2,\alpha_0,\alpha_1,\alpha_2,\beta_0,\beta_1,\beta_2\gamma_0,\gamma_1,\gamma_2,\delta_0,\delta_1,\delta_2\geq 0 \\ & \eta_{11},\xi_{11},\eta_{12},\xi_{12},\eta_{21},\xi_{21},\eta_{22},\xi_{22},p_1,q_1,p_2,q_2\geq 0. \end{array} \tag{19}$$

Optimal solution of this problem using MATLAB is

$$\begin{array}{l} \alpha_1=0.1910,\delta_1=0.2348,\beta_2=0.1495,\gamma_2=0.1838,\alpha_0=0.0077 \\ \delta_0=0.0094,\eta_{11}=0.2196,\eta_{12}=0.3610,\xi_{21}=0.3333,\xi_{22}=0.1668 \\ q_1=0.1999,p_2=0.1667,y_1=1.0254,z=1.2290. \end{array} \tag{20}$$

Using these values in (14), the modified LFP Problem is as follows:

$$\begin{array}{ll} \text{Maximize} & z=\frac{2.1910x_1+2.8505x_2+1.0077}{0.7652x_1+1.1838x_2+3.9906} \\ \text{Subject to ,} & 1.2196x_1+1.3610x_2\leq 3.8001 \\ & 2.6667x_1+0.8333x_2\leq 6.1667 \\ & x_1,x_2\geq 0. \end{array} \tag{21}$$

4. Particular Case

If we substitute $d_j = 0$ for all $j \in J$ and $d_0 = 1$ in original problem, it reduces to linear programming problem and our proposed method reduces for inverse linear programming problem which is studied earlier by Orlin and Ahuja [1].

5. Conclusion

Inverse optimization is an important area in both academic research and practical applications. This paper gives a mathematical model to handle the situations of market when enterprise couldn't response the market demand in time because of rigid treatment of resources. An illustration observation used to demonstrate the advantage of the new approach.

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