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Synthesis of Adequate Mathematical Description as Solution of Special Inverse Problems

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Abstract. The problem of mathematical simulation of dynamic system characteristics behavior and their adequacy to real experimental data, which correspond to these characteristics, is considered in this paper. The specified problem is still poorly investigated and hardly adapted to formalization. The requirements of related to the adequate mathematical simulation of dynamic system are considered for the case when mathematical description is represented by system of the ordinary differential equations. The conditions are obtained which allow to reduce a problem of the adequate mathematical description to the solution of the several integral equations of the first type. The methods of obtaining of the steady solutions are suggested. The domains of application of the obtained solutions are specified. For a case, when the differential equations of dynamic system are given with errors in coefficients, several variants of synthesis of the adequate mathematical descriptions depending on final goals of this description use are suggested. The examples of the adequate descriptions of concrete dynamic systems are given.

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1. Introduction

The problem of synthesis of adequate mathematical description of dynamic system was considered by many authors during the last decades. The contemporary situation in this field has been characterized by R.Shannon [21], who admitted, that despite the extensive literature on a substantiation and study of simulation accuracy, these questions still remain almost so difficult as well as at the beginning of their development. In the synthesis of adequate description of real systems the decisive question is the conformity (in some reasonable sense) of model output to an expected output of real system. In other words, the author specifies importance of a question of adequacy of the mathematical description to a real process.

We consider a typical situation which arises during the analysis of new processes or phenomena. It is supposed that real physical process is observed and the records of experimental

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measurements of some characteristics of this process are given. It is necessary to develop adequate mathematical description of this process for further use.

In this paper the mathematical models of physical processes described only by the system of the ordinary differential equations are examined [1, 9, 19, 20]. Such idealization of real processes or dynamic systems is widely used in various areas for the description of control systems [11], as well as of mechanical systems with the concentrated parameters [2, 7], economic processes [10], biological processes [23], ecological processes [18] etc. In some works with the help of such systems human emotion are simulated [3].

We assume also, that the process is open, i.e. it interacts with other neighboring processes. In the closed processes there is no such interaction.

For the simplicity, we select the dynamic process with mathematical models in the form of linear system of ordinary differential equations:

$$\dot{x}(t) = \tilde{C}x(t) + \tilde{D}z(t), \tag{1}$$

with the equation of observation

$$y(t) = \tilde{F}x(t), \tag{2}$$

where $x(t) = (x_1, x_1, \dots, x_n)^T$ is vector function of variables, which characterizes the state of process ,$z(t) = (z_1, z_1, \dots, z_l)^T$ is vector function of unknown external load, $y(t) = (y_1, \dots, y_m)^T$; $\tilde{C}, \tilde{D}, \tilde{F}$ are matrices with constant coefficients of the appropriate dimension which are given approximately, F is matrix with constant coefficients having dimension $(m \times n)$ and $rankF = m$ ($(\cdot)^T$ is a mark of transposition).

If the part of external loads of real process is known, this case can be reduced to one which is examined before using principle of linearity of initial dynamic system.

We assume that the state variables $x_i(t), 1 \leq i \leq n$ of system (1) correspond to some real characteristics of process which is under investigation $\tilde{x}_i(t), 1 \leq i \leq n$. Let's assume that the vector function $\tilde{y}(t)$ is obtained from experimental measurements.

The *problem of synthesis of adequate mathematical description* with the use of system (1) can be formulated as follows: it is necessary to find unknown vector function of external load $z(t)$ in such a way that the vector function $y(t)$, which are obtained from system (1) under this external load $z(t)$, coincides with experimental data $\tilde{y}(t)$ with accuracy of experimental measurements in chosen functional metrics.

We consider what prospects of adequate descriptions are valid for further use and what goals should be selected in process for creation of adequate mathematical descriptions.

It would be useful to address to classical results in this area. In paper [21] the following statement was done: "... the imitation simulation is the creation of experimental and applied methodology which aimed at the use of it for a prediction of the future behavior of system ".

So the adequate mathematical descriptions are intended for the forecast of behavior of real processes at first. The adequate mathematical simulation helps to predict behavior of real process in new conditions of operation. For example, it is possible to test more intensive mode of operations of the real machine without the risk of its destruction. Such tool (adequate mathematical description) allows to simulate the characteristics of process in the unconventional modes of operations, and also to determine optimum parameters of real process.

The considered situation requires formation of some uniform and methodological approach to this problem, general algorithms, common criteria of evaluation of adequacy degree [14, 16, 21].

The offered research is supposed to have general character and, therefore, it requires analysis of concrete processes in different areas of practice. In this case theoretical results will be valid for wide class of processes.

The given research is devoted to consideration of problems of the specified type. The author hopes that the study of the specified problems will be useful for a construction of the adequate mathematical descriptions of real physical processes.

The paper is organized as follows: definition of problem of synthesis of adequate mathematical description is given in first section, one approach to solution of synthesis problem is presented in second section, the specific features of obtained integral equations and methods of solution are considered in section 3-4, main results (synthesis of adequate mathematical description for some cases) are obtained in section 4,5,6.

2. Statement of Problem of Adequate Mathematical Description Synthesis

There exist two approaches to problem of construction of adequate mathematical description [14, 16]:

i) Mathematical model of process of type (1) is given *a priori* with inexact parameters and then the models of external loads were determined for which the results of mathematical simulation coincide with experiment [14, 16];

ii) Some models of external loads are given a priori and then mathematical model of process of type (1) is chosen for which the results of mathematical simulation coincide with experiment [5, 6, 22].

Now we will consider the synthesis of adequate mathematical description in the frame of first approach if the motion of process is described by ordinary differential equations of n-order (1).

We assume that vector-function $y(t)$ in system (1),(2) is obtained from experiment and presented by graphically.

According to first approach, it is necessary to develop the construction of such vector function of external load which is characterized by the functions of state $x_1(t), x_2(t), \dots, x_n(t)$ of mathematical model (1), and will coincide with experimental measurements $\tilde{y}(t)$ with inaccuracy of initial data in given metrics. Such mathematical model of process behavior together with obtained vector function of external load $z(t)$ can be considered as *adequate mathematical description of process*.

Such method of obtaining of mathematical models of external loads (functions $z_1(t)$, $z_2(t), \dots, z_l(t), l \leq m$) is determined in literature as a method of external loads identification [4, 8]. By the way, physical reasons of occurrence of such external loads are not taken into account. They are only functions which in combination with mathematical model (1) provide results of mathematical simulation, which coincide with experiment with the given accuracy.

Now we consider the synthesis of function of external loads by a method of identification [14, 16].

The part of state variables $\tilde{x}_1(t), \tilde{x}_2(t), \ldots, \tilde{x}_m(t)$ can be obtained by an inverse of equation (2) with function $\tilde{y}(t)$:

$$\tilde{x}_k(t) = N_k(\tilde{y}(t)),$$

where $N_k(\tilde{y}(t))$ is known functions.

Let's consider known state variable $\tilde{x}_k(t)$ as two known internal loads $d_k\tilde{x}_k(t)$ and $[-d_k\tilde{x}_k(t)]$ where d_k is known constants. Such interpretation of state variables $\tilde{x}_k(t)$ allows to simplify initial system. Such transformation will be determined as "k-section" of initial system [14, 16].

In some cases after lines of "k-sections" the initial system (1) will be transformed to some subsystem at which one state variable is known, for example, $\tilde{x}_1(t)$ and at which all external loads $\tilde{z}_k(t), k = 2, \ldots, m$, except $z_1(t)$, for example, are known. For the system of type (2) with the help of a number of "sections" can be obtained the subsystem of initial system which motion parameters are described by the differential equations

$$\dot{x}(t) = A_1 x(t) + B_1 z_1(t), \tag{3}$$

with the equation of observation

$$\tilde{y}_1(t) = c_1 \tilde{x}_1(t), \tag{4}$$

where A_1 is matrix with constant coefficients of the appropriate dimension; B_1 is vector column, c_1 is const. So, it is supposed that for the obtained subsystem one external load is unknown and one variable of state $\tilde{x}_1(t)$ is given.

Motion $x(t) = x(t, t_0, x^0)$ of a subsystem (4) of systems (1) which satisfies the initial condition $x(t_0) = x^0$, are determined by the formula $x(t) = x(t, t_0, x^0)$ [14].

From the equation of observation we will have

$$x_1(t) = c_1^{-1} y_1(t) = \Psi[t - t_0]x^0 + \int_{t_0}^{t} \Psi[t - \tau]B_1 z_1(\tau)d\tau, \tag{5}$$

where $\Psi[t - t_0] = \{\psi_k^{(i)}[t - t_0]\}$ is fundamental matrix of uniform system (3).

The equation (3) gives

$$\begin{aligned} x_1(t) &= \int_{t_0}^{t} \psi_1^{(1)}(t - \tau)b_1 z_1(\tau)d\tau + \sum_{i=1}^{n} \psi_1^{(i)}(t - \tau)x_i^0 \\ &= \sum_{i=1}^{n} \psi_1^{(i)}(t - \tau)x_i^0 + \int_{t_0}^{t} z_1(\tau)\psi_1^{(1)}(t - \tau)b_1 d\tau \\ &= \sum_{i=1}^{n} \psi_1^{(i)}(t - \tau)x_i^0 + \int_{t_0}^{t} K_1(t - \tau)z_1(\tau)d\tau, \end{aligned}$$

where b_1 is the first element of vector column B_1,

$$K_1(t-\tau) = \psi_1^{(1)}(t-\tau)b_1.$$

From (5) we have

$$x_1(t) = \sum_{i=1}^{n} \psi_1^{(i)}(t-\tau)x_i^0 + \int_{t_0}^{t} K_1(t-\tau)z_1(\tau)d\tau$$

or

$$\int_{t_0}^{t} K_1(t-\tau)z_1(\tau)d\tau = P(t), \tag{6}$$

where

$$P(t) = x_1(t) - \sum_{i=1}^{n} \psi_1^{(i)}(t-\tau)x_i^0$$

is known function.

The function $z(t)$ which was obtained with the use of such method depends on chosen mathematical model (1).

If the initial system (1) does not satisfy the condition, as have been specified above, then this system can be reduced to system (3) by aid of additional measurements [14, 16].

With the use of similar transformations it is possible to receive the other integral equations for all unknown functions of external loads $z_k(t)$. After the solving of the integral equations, such as (6), all unknown functions of external loads of system (1) will be obtained.

Further, the function of external loads which are found in such a way together with mathematical model of process (1) gives the adequate mathematical description of process.

Function $P(t)$ in equation (6) was obtained with use of experimental measurements $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n$. So the function $P(t)$ is function which is given as graphic.

Let us present now (6) as

$$A_p z = u_g, \tag{7}$$

where A_p is linear operator, $A_p : Z \to U, z \in Z, u_g \in U$ is initial experimental data (graphic), z is unknown function, $(Z, U$ are functional spaces). Let's assume, that the operator A_p continuously depend on some vector parameters p of mathematical model of process:
$p = (p_1, p_2, \ldots, p_n)^T$. The coefficients of matrices $\tilde{C}, \tilde{D}$ can be chosen as components of such vector parameters p.

It can be shown that in the most practical problems the operator A_p is completely continuous [24].

Thus, a necessary condition for obtaining the equation (7) describing required external load z_j, is the possibility through a number of "sections" to obtain a subsystem of initial system with one unknown external load and one known state variable $x_j, 1 \leq j \leq n$. It is easy to demonstrate an example of system such as (1), in which such opportunity is absent.

3. Specific Features of Integral Equations Solutions

We will consider the integral equation such as (7). From the practical point of view it is convenient to take Z as Banach space of continuous functions $C[0,T]$ or Hilbert space $W_2^1[0,T]$, where $[0,T]$ is interval of time on which the functions of external loads are being investigated [24]. As far as the initial experimental data are frequently varying functions, it is convenient to accept U as Hilbert space $L_2[0,T]$ [24].

Further, we shall suppose that the element u_g in the equation (7) is exchanged by function u_δ which approximated given graphic u_g with a known error:

$$\|u_g - u_\delta\|_U \leq \delta,$$

where δ is const, $\delta > 0$.

Let's denote by $Q_{\delta,p}$ the set of the possible solutions of an inverse problem of identification of function of external load (8) with the fixed operator A_p:

$$Q_{\delta,p} = \{z : \|A_p z - u_\delta\|_U \leq \delta\}.$$

Any function z from set $Q_{\delta,p}$ may be considered as "good" function of external load as far as the function $A_p z$ coincides with u_δ with accuracy of approximation.

Thus, the operators A_p and any function from set $Q_{\delta,p}$ give a pair which will provide adequacy of results of mathematical simulation with accuracy δ.

We shall name the process of determination of $z \in Q_{\delta,p}$ as *synthesis of function of external load by a method of identification* [15].

However set of the possible solutions $Q_{\delta,p}$ at any δ has a number of specific features (it is actually incorrect problem) [24]. First, and main of them is that this set is not bounded at any δ [24].

Let's consider this feature more in detail due to the fact that it leads to a number of unexpected and unusual consequences.

The set $Q_{\delta,p}$ contains infinite number of the solutions like any problem with the use of approximate data. However, the set $Q_{\delta,p}$ contains functions which can differ one from another on infinite value [16, 24]. It is due to the reason that the operator A_p in the equation (8), as a rule, is completely continuous.

Thus, the set $Q_{\delta,p}$ includes the essentially different functions which are equivalent in sense of the solution of the equation (8). Therefore, the basic difficulty will be the selection of the concrete solution from infinite set of the various equivalent solutions. For this purpose it is necessary to involve some additional information [24].

4. Methods of Solution of Identification Equation

For obtaining of the steady solutions of formulated above problems it is necessary to use the method of Tikhonov's regularization [24].

Let us consider the stabilizing functional $\Omega[z]$ which has been defined on set Z_1, where Z_1 is everywhere dense in Z [24]. Consider now the following extreme problem:

$$\Omega[z_{\delta,p}] = \inf_{z \in Q_{\delta,p} \bigcap Z_1} \Omega[z], p \in R^n. \tag{8}$$

Let us assume that the stabilizing quadratic functional $\Omega[z]$ has the additional property (i):

a functional $\Omega[z]$ generates on the set $Z_1 \subset Z$ a Hilbert space $\tilde{Z}_1$ with the majorant norm $\| \cdot \|_{\tilde{Z}_1}$ ($\| \cdot \|_{\tilde{Z}_1} \geq \| \cdot \|_{Z_1}$) [24].

Let the functional $\Omega[z]$ has the additional property (i). It was shown that under these conditions the solution of the extreme problem (8) exists, is unique and stable with respect to small change of initial data u_δ [24]. The function $z_{\delta,p}$ is named *the stable model of external load* after taking into account the only inaccuracy of experimental measurements. The solution of a problem (8) can be non-unique. For the purposes of mathematical simulation any such solution will be acceptable. Such function of external load can be used for mathematical simulation of initial system (1).

Still, there are no basis to believe, that the function $z_{\delta,p}$ will be close to real external load z_{ex}. It is only good and steady function (model) of external load [14, 16]. In addition, the error of function $z_{\delta,p}$ with respect to z_{ex} can be significant. For the purpose of adequate mathematical description synthesis fits any function from the set $Q_{\delta,p}$. This quality of the inverse problem (7) allows us to consider this problem as the special inverse problem.

However, such approximate solution can be interpreted in other way. The regularized solution can be treated as steadiest with respect to change of the factors which were not taken into account in mathematical model. These factors may include changes in structure of mathematical model of system, the influence, which were not taken into account, change of conditions of experiment etc. We can prove such interpretation of the approximate solution.

At the synthesis of mathematical model of physical process we will first of all take into account the factors which define a low-frequency part of change of state variables. First, it is due to the fact that this part of a spectrum is well observed during the experiment, as measuring devices do not deform it. Secondly, the high-frequency components of external loads as well as equivalent to them insignificant factors not taken into account quickly die away in process of distribution among inertial elements. Thus, factors of interactions, which were not taken into account and equivalent to them influences, change only high-frequency part of the approximate solutions. If the factors, which are not taken into account, change a low-frequency part of the solutions then it means that mathematical model of process is chosen incorrectly. In work [25] has been shown that the regularized solution $z_{\delta,p}$ represents result of high-frequency filtration of approximate solution. The greater degree of smoothing of the solution corresponds to greater error of initial data. Hence, the regularized solution $z_{\delta,p}$ can be interpreted as the function from set $Q_{\delta,p}$ which is the steadiest with respect to changes of factors, which are not taken into account. Such quality of the regularized solution is very important when it is used in mathematical simulation of real processes when the results of simulation are steady with respect to small changes factors, which are not taken into account and which are naturally present at any mathematical model of process.

The obtained solution of a synthesis problem of external load function $z_{\delta,p}$ requires, as a rule, the additional analysis. It is necessary, first of all, to determine transformation of model $z_{\delta,p}$ with change of a operational conditions, for example, change of speed of movement of real dynamical system, change of sizes of various static loads etc. As a result of the analysis, the model z_p will be obtained which can be used for mathematical modeling of real processes and also at study of new prospective modes of operations.

For the numerical solution of an extreme problem (8) the discrepancy method was used [24]. The problem (8) was replaced by following extreme problem

$$M^{\alpha}[A_p, z_{\delta,p}] = \inf_{z \in Z_1} M^{\alpha}[A_p, z] = \inf_{z \in Z_1} \{\|A_p z - u_\delta\|_U^2 + \alpha \Omega[z]\}. \tag{9}$$

where parameter of regularization α was determined from a condition

$$\|A_p z_{\delta,p} - u_\delta\|_U = \delta.$$

Theorem 1. *The solution of extreme problem* (9) *exists, unique and is stable to with respect to small change of function u_δ if $\Omega[z]$ is stabilizing functional with additional property (i) [24].*

So the obtained function $z_{\delta,p}$ together with operator A_p give the adequate mathematical description of process which is stable to small change of initial data.

In this case the analysis of a problem was reduced to the solution of the Euler's equation for functional (9) if the functional space U is Hilbert space:

$$A_p^* A_p z_{\delta,p} + \alpha \acute{\Omega}[z_{\delta,p}] = A_p^* u_\delta. \tag{10}$$

where A_p^* is the associate operator to A_p; $\acute{\Omega}[z]$ is Frechet's derivative.

The approximate solution of the equation (10) on a uniform discrete grid was carried out by a numerical method.

5. Synthesis of Adequate Mathematical Description of Main Mechanical Line Dynamics of Rolling Mill

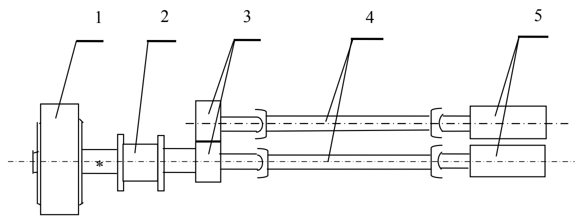
Now we consider in detail the problem of adequate mathematical description synthesis of the main mechanical line of rolling mill [12, 13].

The four-mass model with weightless elastic connections is chosen as mathematical model of dynamic system of the main mechanical line of the rolling mill [12, 13]. One variant of the kinematical scheme of it presented in Figure 1. where: 1 is engine, 2 is coupling, 3 is gears, 4 is driving shafts, 5 is operational barrels.

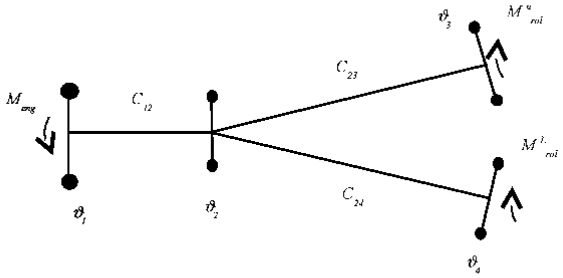
The equations of motion are obtained from the Lagrangian equations of second kind:

$$\begin{aligned} &\ddot{M}_{12} + \omega_{12}^2 M_{12} - \frac{c_{12}}{\vartheta_2}(M_{23} + M_{24}) = \frac{c_{12}}{\vartheta_1} M_{eng}(t), \\ &\ddot{M}_{23} + \omega_{23}^2 M_{23} - \frac{c_{23}}{\vartheta_2} M_{12} + \frac{c_{23}}{\vartheta_3} M_{24} = \frac{c_{23}}{\vartheta_3} M_{rol}^U(t), \end{aligned} \tag{11}$$

$$\ddot{M}_{24} + \omega_{24}^2 M_{24} - \frac{c_{24}}{\vartheta_2} M_{12} + \frac{c_{24}}{\vartheta_4} M_{23} = \frac{c_{24}}{\vartheta_4} M_{rol}^L(t), \quad (12)$$



(a)



(b)

Figure 1: Kinematical scheme of the main mechanical line of rolling mill.

Here the following designations were accepted:M_{eng} is moment of engine, ϑ_i are moments of inertia of the concentrated weights, c_{ik} are rigidity of the appropriate elastic connection, M_{rol}^U, M_{rol}^L are moments of technological resistance put to the upper and lower operational barrels accordingly; M_{ik} are moments of elasticity forces which are applied to shafts between mass ϑ_i and ϑ_k; $\omega_{ik}^2 = c_{ik}(\vartheta_i + \vartheta_i)\vartheta_i^{-1}\vartheta_k^{-1}$.

Actually, the constructed mathematical model may correspond to real process and may not. It is necessary to check up correctness of the constructed mathematical model. For this purpose the data of experiment are used. If the results of mathematical simulation coincide with results of experiment (with accuracy of measurements) then mathematical description of process is considered as adequate to a reality. In other words, the mathematical description corresponds to real process.

The information related to the real motion of the main mechanical line of rolling mill was obtained by an experimental way [12, 13]. Such information is being understood as availability of functions $M_{12}(t), M_{23}(t), M_{24}(t)$. The records of functions $M_{12}(t), M_{23}(t)$, $M_{24}(t)$ of a given process are shown in Figure 2. The errors of experimental measurements are less than to 7% in the uniform metrics.

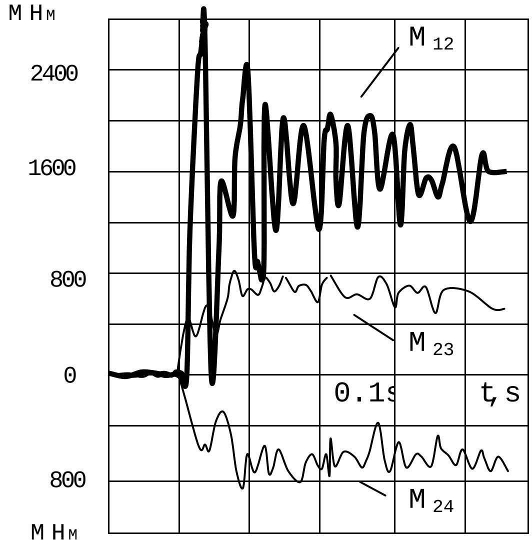


Figure 2: The records of functions $M_{12}(t), M_{23}(t), M_{24}(t)$.

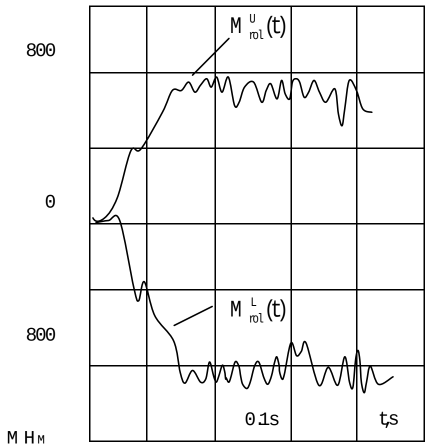


Figure 3: The diagrams of models of external loads $M^U_{rol}(\tau), M^L_{rol}(\tau)$.

It is obvious that the results of mathematical simulation of system (1) depend directly on character of change of external loads which are applied to operational barrels of the rolling mill and external impact of the engine $M_{eng}(t), M^U_{rol}(t), M^L_{rol}(t)$. Sometimes it is possible to pick up such loadings $M_{eng}(t), M^U_{rol}(t), M^L_{rol}(t)$, in which the results of mathematical simulation $M_{12}(t), M_{23}(t), M_{24}(t)$ coincides with experiment (Figure 2). If such choice is possible, then mathematical model (1) combined with the found loads $M_{eng}(t), M^U_{rol}(t), M^L_{rol}(t)$, will give adequate mathematical description of real process.

The problem of the adequacy of mathematical description can be formulated by other way: is it possible to choose the parameters of mathematical model ϑ_i, c_{ik} and its structure in such a manner that the results of mathematical simulation will coincide with experiment with the given accuracy, provided that the certain character of change of external loads $M_{eng}(t), M^U_{rol}(t), M^L_{rol}(t)$ are given. It may be shown that the answer to this question is positive.

The solutions of identification problem of the stable functions of external load on operational barrels, for experimental data presented in Figure 2 was performed by taking into account only inaccuracy of experimental data.

The parameters values of system (11) were selected as follows:

$$C_{12} = 9.5 \cdot 10^5 KHM, C_{23} = C_{24} = 7.5 \cdot 10^4 KHM,$$
$$\vartheta_1 = 110TM^2, \vartheta_2 = 30TM^2, \vartheta_3 = \vartheta_4 = 3.5TM^2.$$

The results of calculation are presented in Figure 3.

6. Synthesis of Adequate Mathematical Description for Class of Operators

It is necessary to apply various methods of simplification in developing the mathematical model, such as taking into account different forces and their impact on the movement of real system. The various models (with different parameters) of real process or system were obtained by the different authors even in cases, when the structures of the mathematical descriptions (models) are similar. So, it is supposed that the vector parameters p is given inexactly. So vector p can get different values in given closed domain $D : p \in D \subset R^n$.

Some operator A_p corresponds to each vector from D. The set of possible operators A_p has been denoted as class of operators $K_A = \{A_p\}$. So we have $A_p \in K_A$. The deviations of operators $A_p \in K_A$ between themselves from set K_A can be evaluated as:

$$\sup_{p_\alpha, p_\beta \in D} \|A_{p_\alpha} - A_{p_\beta}\|_{Z \to U} \leq h_1.$$

Now we transfer to consideration of a more general problem of synthesis of external loads functions in which the inaccuracy of operator A_p will be taken into account [17].

The set of possible solution of equation (8) is necessary to extend to set $Q_{h_1,\delta}$ taking into account the inaccuracy of the operator $A_p, p \in D$ [24]:

$$Q_{h_1,\delta} = \{z : \|A_p z - u_\delta\|_U \leq h_1 \|z\|_Z + \delta\}.$$

Any function from $Q_{h_1,\delta}$ causes the response of mathematical model coinciding with the response of investigated object with an error into which errors of experimental measurements and errors of a possible deviation of parameters of a vector $p \in D$ are included.

It should be noted that the set of the solutions of a problem of synthesis $Q_{h_1,\delta}$ at the fixed operator A_p from K_A in $Q_{h_1,\delta}$ contain elements with unlimited norm (incorrect problem) therefore the value $h_1\|z\|_Z + \delta$ can be infinitely large. Formally speaking, such situation is unacceptable as it means that the error of mathematical simulation tends to infinity, if for the simulation of external load is used the arbitrary function from $Q_{h_1,\delta}$ as functions of external load. Hence not all functions from $Q_{h_1,\delta}$ will serve as "good" functions of external load.

Another method of accounting errors operator $A_p \in K_A$ in equation (7) is proposed to overcome this limitation. The following set Q_δ^* is using instead of the set of possible solutions $Q_{h_1,\delta}$.

$$Q_\delta^* = \bigcup_{A_p \in K_A} Q_{\delta,p},$$

where $\bigcup$ is the sign of union.

Any function of the set Q_δ^* satisfies equation (7) up with the inaccuracy δ for the corresponding operator $A_p \in K_A$.

Any function from Q_δ^* in conjunction with the corresponding operator A_p gives a couple which provides an adequate description of the physical process.

The functions of external load $z(t)$ in this case can be different. They will depend on final goals of mathematical simulation.

Let us considered some extreme problems which may be investigated in this case:

$$\Omega[z_0] = \inf_{z \in Q_\delta^*} \Omega[z] = \inf_{A_p \in K_A} \inf_{z \in Q_{\delta,p} \bigcap Z_1} \Omega[z]. \tag{13}$$

$$\Omega[z^1] = \sup_{A_p \in K_A} \inf_{z \in Q_{\delta,p} \bigcap Z_1} \Omega[z]. \tag{14}$$

Solution z_0 of the extreme problem (13) is a model of external load which is stable to small changes of the initial data.

Function $A_{p_0} z_0$ coincides with the given function u_g with the inaccuracy δ. From a practical point of view the function z_0 allows you to answer the question under what parameters the minimum of functional $\Omega[z]$ on the set Q_δ^* achieves, under what parameters will be achieved a minimum of energy control, for example. Similar properties will have the solution z^1. From a practical point of view the function z^1 allows you to answer the question under what parameters the maximum of functional $\Omega[z]$ on the set Q_δ^* reaches, under what parameters will be reached a maximum of energy control, for example.

Theorem 2. *The solution of extreme problem* (13) z_0 *exists, has a unique and stable solution to small changes of initial data if the stabilizing quadratic functional* $\Omega[z]$ *has the additional property (i) and the function* $\varphi(p) = \Omega[z_{\delta,p}]$ *is a strongly convex on a set D for any* $\delta > 0$ *[17].*

Two couples $\{A_{p_0}, z_0\}$ and $\{A_{p^1}, z^1\}$ fit for the purpose of adequate mathematical description synthesis.

The function of external load, which is necessary for estimation from below of output of dynamic system (process) can be obtained as solution of the following extreme problem [14, 16]:

$$\|A_{p_b} z_b\|_U = \inf_{p \in D} \|A_p z_{\delta,p}\|_U, \tag{15}$$

where $z_{\delta,p}$ is the solution of extreme problem (8).

Another model for estimation from above of output of dynamic system (process) can be obtained as solution the extreme problem [14, 15]:

$$\|A_{p_c} z_c\|_U = \sup_{p \in D} \|A_p z_{\delta,p}\|_U. \tag{16}$$

As unitary model z_{un} we can call the solution of following extreme problem [14, 16]:

$$\|A_{p_{un}} z_{un} - u_\delta\|_U = \inf_{z_{\delta.a}} \sup_{p \in D} \|A_p z_{\delta,a} - u_\delta\|_U, \tag{17}$$

where $z_{\delta,a}$ is the solution of extreme problem (8) with $p = a, a \in D$.

Let us suggest that function $z_{g,p} \in Q_{\delta,p}$ bounded in Z for any $p \in D(A_p z_{g,p} = u_g)$. By Q_p denote the set of functions $z_{g,p}$: $Q_p = \{z_{g,p}\}$.

Now we will consider additional extreme problem

$$\|A_{p_0} \hat{z}_0 - u_g\|_U = \inf_{z \in Q_p} \sup_{A_{p_\alpha} \in K_A} \|A_{p_\alpha} z - u_g\|_U, p_\alpha, p \in D, u_g \in E(A_p). \tag{18}$$

So the pair $\{A_{p_{un}}, z_{un}\}$ gives the stable adequate mathematical description for class of operators of process as example of possible one.

A problem of a finding $z \in Q^*_\delta$ will be entitled by analogy to the previous one as a *problem of synthesis for a class of operators* [15].

The method of special mathematical model selection was suggested for solution of such extreme problems [16].

The real calculations of external loads function z_{un} on rolling mill were presented as an example in [12, 15].

The variations of parameters are chosen as follows:

$$9.0 \cdot 10^5 KHM \le C_{12} \le 1.0 \cdot 10^6 KHM, 7.5 \cdot 10^4 KHM \le C_{23} = C_{24} \le 7.5 \cdot 10^4 KHM,$$
$$100TM^2 \le \vartheta_1 \le 120TM^2, 25TM^2 \le \vartheta_2 \le 35TM^2, 3TM^2 \le \vartheta_3 \le \vartheta_4 \le 4.0TM^2.$$

For the case which is shown on Figure 2. the function of external load as solution of extreme problem (16) is presented on Figure 4.

It is necessary to note that results of synthesis do not vary if to change the initial data within the limits of accuracy of measurements δ and if to change the initial dynamic system, so that the inaccuracy of operator A_p would not differ from any operator $A_{p_\alpha} \in K_A$ on value less then h_1 accordingly.

The results of calculations show that the estimation from above of accuracy of mathematical modeling with model z_{un} for all $A_p \in K_A$ does not exceed 11% in the uniform metrics with average error of mathematical model parameters of the main mechanical line of rolling mill equal to 10% and errors of experimental measurements equal to 7% in the uniform metrics.

Note that taking into account the error of the operator A_p leads to more smooth results of identification (see Figure 2. and Figure 4).

The comparative analysis of mathematical modeling with various known models of external loads and experimental data were presented in work [13]. The model of external load z_{un} better corresponds to new experimental observations.

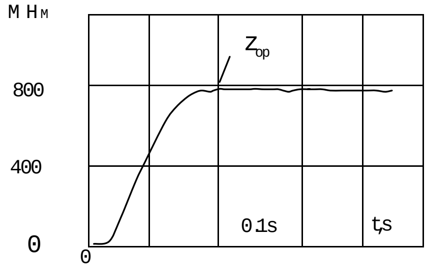


Figure 4: The diagrams of models change of the technological resistance moment on the upper worker barrel of rolling mill.

7. Conclusion

The problems of synthesis of adequate mathematical description of real dynamical system are considered in this work. One of the possible solutions of above-mentioned problems is the choice of function of external loads adapted to dynamical system by identification method. The peculiarities of such approach were investigated. These problems are actually incorrect ones by their nature and that is way for their solution were used the regularization Tikhonov's method. For the case when mathematical model are given approximately, different variants of choice of external loads functions which depend on final goals of mathematical simulation are considered. It can be as follows: simulation of given motion of system, different estimation of responses of dynamic system, simulation of best forecast of system motion, the most stable model with respect to small change of initial data, unitary model etc.

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References

[1] M. Alexik. Modelling and Identification of Eye-Hand Dynamics. Simulation Practice and Theory, 8, 25-38, 2000.

[2] J. Awrejcewicz and V. Krysko. Introduction to Asymptotic Methods. Taylor and Francis, 2006.

[3] F. Breitenecker, F. Judex, N. Popper, K. Breitenecker, A. Mathe and S. Wassertheurer. Laura and Petrarca - True Emotions vs. Modelled Emotions. 6-th Vienna Conference on mathematical Modelling, Vienna, full Papers CD Volume, Vienna Univ. of Technology, ISBM 978-3-901608-35-3, 46-69, 2009.

[4] Ju. Gelfandbein and L. Kolosov. Retrospective Identification of Perturbations and Interferences. moscow, Science, 1972.

[5] V. Gubarev. Method of Iterative Identification of Many-Dimensional Systems with Inexact Data. Part 1. Theoretical basises. Problems of Control and Information, Kiev, Ukraine, 2, 8-26, 2008.

[6] O. Gukov. Algorithms of Iterative Identification of Many-Dimensional Systems. XV International Conference on Authomatical Control "Authomatics -2008", Odessa: INI, Ukraine, 774-777, 2008.

[7] H. Hirahara. Engine Modeling and Control System Design Considering Twist of a Crank Shaft. 6-th Vienna Conference on mathematical Modelling, Vienna - Full Papers CD Volume, Vienna Univ. of Technology, ISBM 978-3-901608-35-3, 173-179, 2009.

[8] S. Ikeda, S. Migamoto, Y. Sawaragi. Regularization Method for Identification of Distributed Systems. IY a Symposium IFAC, Identification and Evaluation of Parameters of Systems, Tbilisi, USSR, Preprint. moscow, 3, 153-162, 1976.

[9] N. Krasovskij. Theory of Motion Control. Science, Moscow, 1968.

[10] V. C. Krass and B. P. Chuprinin. Shape Mathematics in Economics. Mathematical Methods and Models, Finance and Statistics, Moscow, 2007.

[11] Y. Liu, D. Soffker. Robust Control Approach for Input-Output Linearizable Nonlinear Systems with Modeling Errors Based on High-Gain PI-Observer. 6-th Vienna Conference on mathematical Modelling, Vienna, full Papers CD Volume, Vienna Univ. of Technology, ISBM 978-3-901608-35-3, 193-199, 2009.

[12] Yu. Menshikov. Identification of Moment of Technological Resistance on Rolling Mill. Journal of Differential Equations and Their Applications in Physics, Dnepropetrovsk University, Ukraine, 1, 22-28, 1976.

[13] Yu. Menshikov. The Models of External Actions for Mathematical Simulation. System Analysis and Mathematical Simulation, 14(2), 139-147, 2004.

[14] Yu. Menshikov. About Adequacy of Mathematical Modeling Results. International Conference "Simulation-2008", Kiev, Ukraine, 119-124, 2008.

[15] Yu. Menshikov. Robotics, Automation and Control. Chapter 7. The Identification of Models of External Loads, In-Teh is Groatian Branch of I-Tech and Publishing KG, Vienna, Austria, 2008.

[16] Yu. Menshikov. Algorithms of Construction of Adequate Mathematical Description of Dynamic System. 6-th Vienna Conference on mathematical Modelling, Vienna - Full Papers CD Volume, Vienna Univ. of Technology, ISBM 978-3-901608-35-3, 2482-2485, 2009.

[17] Yu. Menshikov. Inverse Problems in Non-classical Statements. International Journal of Pure and Applied Mathematics, 67(1), 79-96, 2011.

[18] V. Perminov. Mathematical Modeling of Large Forest Fires Initiation. 6-th Vienna Conference on Mathematical Modelling, Vienna, full Papers CD Volume, Vienna Univ. of Technology, ISBM 978-3-901608-35-3, 1165-1172, 2009.

[19] W. Porter. Modern Foundations of Systems Engineering. The Macmillan Company, New York, Collier-Macmillan Limited, London, 1970.

[20] I. Sarmar and A. Malik. Modeling, Analysis and Simulation of a Pan Tilt Platform Based on Linear and Nonlinear Systems. IEEE/ASME MESA, China, 147-152, 2008.

[21] R. Shannon. Systems Simulation - The Art and Science. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1975.

[22] V. Stepashko. Method of Critical Dispersions as Analytical Apparatus of Theory of Inductive Modeling. Problems of Control and Information, Kiev, Ukraine, 2, 27-32, 2008.

[23] J. Tillack, S. Noack, K. Noh, A. Elsheikh and W. Wiechert. A Software Framework for Modeling and Simulation of Dynamic Metabolic and Isotopic Systems. 6-th Vienna Conference on Mathematical Modelling, Vienna, full Papers CD Volume, Vienna Univ. of Technology, ISBM 978-3-901608-35-3, 769-778, 2009.

[24] A.Tikhonov and V.Arsenin. Methods of Solution of Incorrectly Problems. Science, Moscow, 1979.

[25] S. Vilenkin. Application of Regularization for Evaluation of Input Signal under Realization of Output Signal. Automation and Telemechanics, 21, 52-55, 1968.