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On Almost γ-continuous Functions

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Abstract. We introduce a new class of functions called almost γ-continuous functions which is contained in the class of almost continuous functions and contains the class of γ-continuous functions.

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1. Introduction

Kasahara [9] defined an operation α on a topological space to introduce α-closed graphs. Following the same technique, Ogata [16] defined an operation γ on a topological space and introduced γ-open sets. Basu et al. [4] introduced a type of continuity called γ-continuous function. Singal and Singal [18] introduced the notion of almost continuity.

In this paper, we introduce a new class of functions called almost γ-continuous functions which is contained in the class of almost continuous functions and contains the class of γ-continuous functions. We obtain basic properties of almost γ-continuous functions.

2. Preliminaries

Throughout this paper, (X,τ) and (Y,σ) stand for topological spaces with no separation axioms assumed unless otherwise stated. For a subset A of X, the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively. Let (X,τ) be a space and A a subset of X. An operation γ [16] on a topology τ is a mapping from τ in to power set $P(X)$ of X such that $V \subseteq \gamma(V)$ for each $V \in \tau$, where $\gamma(V)$ denotes the value of γ at V. A subset A of X with an operation γ on τ is called γ-open [16] if for each $x \in A$, there exists an open set U such that $x \in U$ and $\gamma(U) \subseteq A$. Then, τ_γ denotes the set of all γ-open set in X. Clearly $\tau_\gamma \subseteq \tau$. Complements of γ-open sets are called γ-closed. The τ_γ-interior [10] of A is denoted by τ_γ-$Int(A)$ and defined to be the union of all γ-open sets of X contained in A. A topological X with an operation γ on τ is said to be γ-regular [16] if for each

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$x \in X$ and for each open neighborhood V of x, there exists an open neighborhood U of x such that $\gamma(U)$ contained in V. It is also to be noted that $\tau = \tau_\gamma$ if and only if X is a γ-regular space [16].

Definition 1. *A subset A of a space X is said to be*

(i) *α-open [14] if $A \subseteq Int(Cl(Int(A)))$.*

(ii) *semi-open [12] if $A \subseteq Cl(Int(A))$.*

(iii) *preopen [13] if $A \subseteq Int(Cl(A))$.*

(iv) *β-open [1] if $A \subseteq Cl(Int(Cl(A)))$.*

Definition 2. *The intersection of all preclosed (resp., semi-closed, α-closed) sets of X containing A is called the preclosure [7] (resp., semi-closure [5], α-closure [17]) of A.*

Definition 3 ([19]). *The δ-interior of a subset A of X is the union of all regular open sets of X contained in A. The subset A is called δ-open if $A = Int_\delta(A)$, i.e. a set is δ-open if it is the union of regular open sets. The complement of a δ-open set is called δ-closed. Alternatively, a set $A \subseteq X$ is called δ-closed if $A = Cl_\delta(A)$, where*
$Cl_\delta(A) = \{x \in X : Int(Cl(U)) \cap A \neq \phi, U \in \tau \text{ and } x \in U\}$.

Proposition 1 ([2]). *A subset A of a space X is β-open if and only if $Cl(A)$ is regular closed.*

Theorem 1 ([1]). *Let A be any subset of a space X. Then $A \in \beta O(X)$ if and only if $Cl(A) = Cl(Int(Cl(A)))$.*

Theorem 2. *Let A be a subset of a topological space (X, τ). Then:*

(i) *If $A \in SO(X)$, then $pCl(A) = Cl(A)$ [6].*

(ii) *If $A \in \beta O(X)$, then $\alpha Cl(A) = Cl(A)$ [3].*

(iii) *If $A \in \beta O(X)$, then $Cl_\delta(A) = Cl(A)$ [20].*

Lemma 1 ([8]). *Let A be a subset of a space (X, τ). Then $A \in PO(X, \tau)$ if and only if $sCl(A) = Int(Cl(A))$.*

Proposition 2 ([11]). *Let A be any subset of a topological space (X, τ) and γ be an operation on τ. Then the following statements are true:*

(i) *$X \setminus \tau_\gamma - Int(A) = \tau_\gamma - Cl(X \setminus A)$.*

(ii) *$X \setminus \tau_\gamma - Cl(A) = \tau_\gamma - Int(X \setminus A)$.*

Definition 4 ([4]). *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be γ-continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a γ-open set U containing x such that $f(U) \subseteq V$.*

Definition 5 ([18]). *A function $f : (X,\tau) \to (Y,\sigma)$ is called almost continuous at $x \in X$ if for every open set V in Y containing $f(x)$, there exists an open set U in X containing x such that $f(U) \subseteq Int(Cl(V))$. If f is almost continuous at every point of X, then it is called almost continuous.*

Definition 6 ([15]). *A space X is said to be semi-regular if for any open set U of X and each point $x \in U$, there exists a regular open set V of X such that $x \in V \subseteq U$.*

3. Almost γ-Continuous

Definition 7. *A function $f : (X,\tau) \to (Y,\sigma)$ is called almost γ-continuous at a point $x \in X$ if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a γ-open set U of X containing x such that $f(U) \subseteq Int(Cl(V))$. If f is almost γ-continuous at every point of X, then it is called almost γ-continuous.*

Remark 1. *It easily follows that γ-continuity implies almost γ-continuity and almost γ-continuity implies almost continuity. However, the converses are not true as the following example show.*

Example 1. *Consider $X = \{a,b,c\}$ with the topology $\tau = \sigma = \{\phi,\{a\},\{b\},\{a,b\},\{a,c\},X\}$. Define an operation γ on τ by $\gamma(A) = A$ for all $A \in \tau$. Define a function $f : (X,\tau) \to (X,\sigma)$ as follows:*

$$f(x) = \begin{cases} c & \text{if } x = a \\ b & \text{if } x = b \\ a & \text{if } x = c \end{cases}$$

Then f is almost γ-continuous but not γ-continuous, because $\{a\}$ is an open set in (X,σ) containing $f(c) = a$, but there exist no γ-open set U in (X,τ) containing c such that $f(U) \subseteq \{a\}$.
And we define an operation γ on τ by $\gamma(A) = X$ for all $A \in \tau$. Then f is almost continuous which is not almost γ-continuous.

Theorem 3. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following statements are equivalent:*

(i) *f is almost γ-continuous.*

(ii) *For each $x \in X$ and each open set V of Y containing $f(x)$, there exists a γ-open set U in X containing x such that $f(U) \subseteq sCl(V)$.*

(iii) *For each $x \in X$ and each regular open set V of Y containing $f(x)$, there exists a γ-open set U in X containing x such that $f(U) \subseteq V$.*

(iv) *For each $x \in X$ and each δ-open set V of Y containing $f(x)$, there exists a γ-open set U in X containing x such that $f(U) \subseteq V$.*

Proof. (1) $\Rightarrow$ (2). Let $x \in X$ and Let V be any open set of Y containing $f(x)$. By (1), there exists a γ-open set U of X containing x such that $f(U) \subseteq Int(Cl(V))$. Since V is open and hence V is preopen set. By Lemma 1, $Int(Cl(V)) = sCl(V)$. Therefore, $f(U) \subseteq sCl(V)$.
(2) $\Rightarrow$ (3). Let $x \in X$ and Let V be any regular open set of Y containing $f(x)$. Then V is an open set of Y containing $f(x)$. By (2), there exists a γ-open set U in X containing x such that $f(U) \subseteq sCl(V)$. Since V is regular open and hence is preopen set. By Lemma 1, $sCl(V) = Int(Cl(V))$. Therefore, $f(U) \subseteq Int(Cl(V))$. Since V is regular open, then $f(U) \subseteq V$.
(3) $\Rightarrow$ (4). Let $x \in X$ and Let V be any δ-open set of Y containing $f(x)$. Then for each $f(x) \in V$, there exists an open set G containing $f(x)$ such that $G \subseteq Int(Cl(G)) \subseteq V$. Since $Int(Cl(G))$ is regular open set of Y containing $f(x)$. By (3), there exists a γ-open set U in X containing x such that $f(U) \subseteq Int(Cl(G)) \subseteq V$. This completes the proof.
(4) $\Rightarrow$ (1). Let $x \in X$ and Let V be any open set of Y containing $f(x)$. Then $Int(Cl(V))$ is δ-open set of Y containing $f(x)$. By (4), there exists a γ-open set U in X containing x such that $f(U) \subseteq Int(Cl(V))$. Therefore, f is almost γ-continuous.

Theorem 4. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

(i) f is almost γ-continuous.

(ii) $f^{-1}(Int(Cl(V)))$ is γ-open set in X, for each open set V in Y.

(iii) $f^{-1}(Cl(Int(F)))$ is γ-closed set in X, for each closed set F in Y.

(iv) $f^{-1}(F)$ is γ-closed set in X, for each regular closed set F of Y.

(v) $f^{-1}(V)$ is γ-open set in X, for each regular open set V of Y.

Proof. (1) $\Rightarrow$ (2). Let V be any open set in Y. We have to show that $f^{-1}(Int(Cl(V)))$ is γ-open set in X. Let $x \in f^{-1}(Int(Cl(V)))$. Then $f(x) \in Int(Cl(V))$ and $Int(Cl(V))$ is a regular open set in Y. Since f is almost γ-continuous. Then by Theorem 3, there exists a γ-open set U of X containing x such that $f(U) \subseteq Int(Cl(V))$. Which implies that $x \in U \subseteq f^{-1}(Int(Cl(V)))$. Therefore, $f^{-1}(Int(Cl(V)))$ is γ-open set in X.
(2) $\Rightarrow$ (3). Let F be any closed set of Y. Then $Y \setminus F$ is an open set of Y. By (2), $f^{-1}(Int(Cl(Y \setminus F)))$ is γ-open set in X and

$$f^{-1}(Int(Cl(Y \setminus F))) = f^{-1}(Int(Y \setminus Int(F))) = f^{-1}(Y \setminus Cl(Int(F))) = X \setminus f^{-1}(Cl(Int(F)))$$

is γ-open set in X and hence $f^{-1}(Cl(Int(F)))$ is γ-closed set in X.
(3) $\Rightarrow$ (4). Let F be any regular closed set of Y. Then F is a closed set of Y. By (3), $f^{-1}(Cl(Int(F)))$ is γ-closed set in X. Since F is regular closed set. Then
$f^{-1}(Cl(Int(F))) = f^{-1}(F)$. Therefore, $f^{-1}(F)$ is γ-closed set in X.
(4) $\Rightarrow$ (5). Let V be any regular open set of Y. Then $Y \setminus V$ is regular closed set of Y and by (4), we have $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is γ-closed set in X and hence $f^{-1}(V)$ is γ-open set in X.
(5) $\Rightarrow$ (1). Let $x \in X$ and let V be any regular open set of Y containing $f(x)$. Then

$x \in f^{-1}(V)$. By (5), we have $f^{-1}(V)$ is γ-open set in X. Therefore, we obtain $f(f^{-1}(V)) \subseteq V$. Hence by Theorem 3, f is almost γ-continuous.

Theorem 5. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

(i) f *is almost γ-continuous.*

(ii) $f(\tau_\gamma\text{-}Cl(A)) \subseteq Cl_\delta(f(A))$, *for each subset A of X.*

(iii) $\tau_\gamma\text{-}Cl(f^{-1}(B)) \subseteq f^{-1}(Cl_\delta(B))$, *for each subset B of Y.*

(iv) $f^{-1}(F)$ *is γ-closed set in X, for each δ-closed set F of Y.*

(v) $f^{-1}(V)$ *is γ-open set in X, for each δ-open set V of Y.*

(vi) $f^{-1}(Int_\delta(B)) \subseteq \tau_\gamma\text{-}Int(f^{-1}(B))$, *for each subset B of Y.*

(vii) $Int_\delta(f(A)) \subseteq f(\tau_\gamma\text{-}Int(A))$, *for each subset A of X.*

Proof. (1) ⇒ (2). Let A be a subset of X. Since $Cl_\delta(f(A))$ is δ-closed set in Y, it is denoted by $\cap\{F_\alpha : F_\alpha \in RC(Y), \alpha \in \Delta\}$, where Δ is an index set. Then, we have $A \subseteq f^{-1}(Cl_\delta(f(A))) = f^{-1}(\cap\{F_\alpha : \alpha \in \Delta\}) = \cap\{f^{-1}(F_\alpha) : \alpha \in \Delta\}$. By (1) and Theorem 4, $f^{-1}(Cl_\delta(f(A)))$ is γ-closed set of X. Hence $\tau_\gamma\text{-}Cl(A) \subseteq f^{-1}(Cl_\delta(f(A)))$. Therefore, we obtain $f(\tau_\gamma\text{-}Cl(A)) \subseteq Cl_\delta(f(A))$.
(2) ⇒ (3). Let B be any subset of Y. Then $f^{-1}(B)$ is a subset of X. By (2), we have $f(\tau_\gamma - Cl(f^{-1}(B))) \subseteq Cl_\delta(f(f^{-1}(B))) = Cl_\delta(B)$. Hence $\tau_\gamma\text{-}Cl(f^{-1}(B)) \subseteq f^{-1}(Cl_\delta(B))$.
(3) ⇒ (4). Let F be any δ-closed set of Y. By (3), we have $\tau_\gamma - Cl(f^{-1}(F))) \subseteq f^{-1}(Cl_\delta(F)) = f^{-1}(F)$ and hence $f^{-1}(F)$ is γ-closed set in X.
(4) ⇒ (5). Let V be any δ-open set of Y. Then $Y \setminus V$ is δ-closed set of Y and by (4), we have $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$ is γ-closed set in X. Hence $f^{-1}(V)$ is γ-open set in X.
(5) ⇒ (6). For each subset B of Y. We have $Int_\delta(B) \subseteq B$. Then $f^{-1}(Int_\delta(B)) \subseteq f^{-1}(B)$. By (5), $f^{-1}(Int_\delta(B))$ is γ-open set in X. Then $f^{-1}(Int_\delta(B)) \subseteq \tau_\gamma\text{-}Int(f^{-1}(B))$.
(6) ⇒ (7). Let A be any subset of X. Then $f(A)$ is a subset of Y. By (6), we obtain that $f^{-1}(Int_\delta(f(A))) \subseteq \tau_\gamma\text{-}Int(f^{-1}(f(A)))$. Hence $f^{-1}(Int_\delta(f(A))) \subseteq \tau_\gamma\text{-}Int(A)$. Which implies that $Int_\delta(f(A)) \subseteq f(\tau_\gamma\text{-}Int(A))$.
(7) ⇒ (1). Let $x \in X$ and V be any regular open set of Y containing $f(x)$. Then $x \in f^{-1}(V)$ and $f^{-1}(V)$ is a subset of X. By (7), we get $Int_\delta(f(f^{-1}(V))) \subseteq f(\tau_\gamma\text{-}Int(f^{-1}(V)))$ implies that $Int_\delta(V) \subseteq f(\tau_\gamma\text{-}Int(f^{-1}(V)))$. Since V is regular open set and hence is δ-open set, then $V \subseteq f(\tau_\gamma\text{-}Int(f^{-1}(V)))$ this implies that $f^{-1}(V) \subseteq \tau_\gamma\text{-}Int(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is γ-open set in X which contains x and clearly $f(f^{-1}(V)) \subseteq V$. Hence, by Theorem 3, f is almost γ-continuous.

Theorem 6. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(i) f *is almost γ-continuous.*

(ii) $\tau_\gamma\text{-}Cl(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$, *for each β-open set V of Y.*

(iii) $f^{-1}(Int(F)) \subseteq \tau_\gamma\text{-}Int(f^{-1}(F))$, *for each* β*-closed set* F *of* Y.

(iv) $f^{-1}(Int(F)) \subseteq \tau_\gamma\text{-}Int(f^{-1}(F))$, *for each semi-closed set* F *of* Y.

(v) $\tau_\gamma\text{-}Cl(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$, *for each semi-open set* V *of* Y.

Proof. (1) ⇒ (2). Let V be any β-open set of Y. It follows from Proposition 1, that $Cl(V)$ is regular closed set in Y. Since f is almost γ-continuous. Then by Theorem 4, $f^{-1}(Cl(V))$ is γ-closed set in X. Therefore, we obtain $\tau_\gamma\text{-}Cl(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$.
(2) ⟺ (3). Let F be any β-closed set of Y. Then $Y \setminus F$ is β-open set of Y and by (2), we have

$$\tau_\gamma - Cl(f^{-1}(Y \setminus F)) \subseteq f^{-1}(Cl(Y \setminus F)) \Leftrightarrow \tau_\gamma - Cl(X \setminus f^{-1}(F)) \subseteq f^{-1}(Y \setminus Int(F)) \Leftrightarrow X \setminus \tau_\gamma - Int(f^{-1}(F)) \subseteq X \setminus f^{-1}(Int(F)).$$

Therefore, $f^{-1}(Int(F)) \subseteq \tau_\gamma\text{-}Int(f^{-1}(F))$.
(3) ⇒ (4). This is obvious since every semi-closed set is β-closed set.
(4) ⇒ (5). Let V be any semi-open set of Y. Then $Y \setminus V$ is semi-closed set and by (4), we have

$$f^{-1}(Int(Y \setminus V)) \subseteq \tau_\gamma - Int(f^{-1}(Y \setminus V)) \Leftrightarrow f^{-1}(Y \setminus Cl(V)) \subseteq \tau_\gamma - Int(X \setminus f^{-1}(V)) \Leftrightarrow X \setminus f^{-1}(Cl(V)) \subseteq X \setminus \tau_\gamma - Cl(f^{-1}(V)).$$

Therefore, $\tau_\gamma\text{-}Cl(f^{-1}(V)) \subseteq f^{-1}(Cl(V))$.
(5) ⇒ (1). Let F be any regular closed set of Y. Then F is semi-open set of Y. By (5), we have $\tau_\gamma\text{-}Cl(f^{-1}(F)) \subseteq f^{-1}(Cl(F)) = f^{-1}(F)$. This shows that $f^{-1}(F)$ is γ-closed set in X. Therefore, by Theorem 4, f is almost γ-continuous.

Theorem 7. *For a function* $f : (X, \tau) \rightarrow (Y, \sigma)$ *with* γ*-regular operation* γ *on* τ*, the following statements are equivalent:*

(i) f *is almost* γ*-continuous.*

(ii) $\tau_\gamma\text{-}Cl(f^{-1}(V)) \subseteq f^{-1}(\alpha Cl(V))$, *for each* β*-open set* V *of* Y.

(iii) $\tau_\gamma\text{-}Cl(f^{-1}(V)) \subseteq f^{-1}(Cl_\delta(V))$, *for each* β*-open set* V *of* Y.

(iv) $\tau_\gamma\text{-}Cl(f^{-1}(V)) \subseteq f^{-1}(\tau_\gamma\text{-}Cl(V))$, *for each semi-open set* V *of* Y.

(v) $\tau_\gamma\text{-}Cl(f^{-1}(V)) \subseteq f^{-1}(pCl(V))$, *for each semi-open set* V *of* Y.

Proof. (1) ⇒ (2). Follows from Theorem 6 and Theorem 2 (2).
(2) ⇒ (3). This is obvious since $\alpha Cl(V) \subseteq Cl_\delta(V)$ in general.
(3) ⇒ (4) and (4) ⇒ (5). Follows from Theorem 2 and the fact $\tau = \tau_\gamma$.
(5) ⇒ (1). Follows from Theorem 6 and Theorem 2 (1).

Corollary 1. *For a function* $f : X \rightarrow Y$ *with* γ*-regular operation* γ *on* τ*, the following statements are equivalent:*

(i) f *is almost* γ*-continuous.*

(ii) $f^{-1}(\alpha Int(F)) \subseteq \tau_\gamma\text{-}Int(f^{-1}(F))$*, for each* β*-closed set* F *of* Y*.*

(iii) $f^{-1}(Int_\delta(F)) \subseteq \tau_\gamma\text{-}Int(f^{-1}(F))$*, for each* β*-closed set* F *of* Y*.*

(iv) $f^{-1}(\tau_\gamma\text{-}Int(F)) \subseteq \tau_\gamma\text{-}Int(f^{-1}(F))$*, for each semi-closed set* F *of* Y*.*

(v) $f^{-1}(pInt(F)) \subseteq \tau_\gamma\text{-}Int(f^{-1}(F))$*, for each semi-closed set* F *of* Y*.*

Theorem 8. *A function* $f : X \to Y$ *is almost* γ*-continuous if and only if* $f^{-1}(V) \subseteq \tau_\gamma\text{-}Int(f^{-1}(Int(Cl(V))))$ *for each preopen set* V *of* Y*.*

Proof. **Necessity.** Let V be any preopen set of Y. Then $V \subseteq Int(Cl(V))$ and $Int(Cl(V))$ is regular open set in Y. Since f is almost γ-continuous, by Theorem 4, $f^{-1}(Int(Cl(V)))$ is γ-open set in X and hence we obtain that $f^{-1}(V) \subseteq f^{-1}(Int(Cl(V))) = \tau_\gamma\text{-}Int(f^{-1}(Int(Cl(V))))$.
Sufficiency. Let V be any regular open set of Y. Then V is preopen set of Y. By hypothesis, we have $f^{-1}(V) \subseteq \tau_\gamma\text{-}Int(f^{-1}(Int(Cl(V)))) = \tau_\gamma\text{-}Int(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is γ-open set in X and hence by Theorem 4, f is almost γ-continuous.

Corollary 2. *A function* $f : X \to Y$ *is almost* γ*-continuous if and only if* $f^{-1}(V) \subseteq \tau_\gamma\text{-}Int(f^{-1}(sCl(V)))$ *for each preopen set* V *of* Y*.*

Corollary 3. *A function* $f : X \to Y$ *is almost* γ*-continuous if and only if* $\tau_\gamma\text{-}Cl(f^{-1}(Cl(Int(F)))) \subseteq f^{-1}(F)$ *for each preclosed set* F *of* Y*.*

Corollary 4. *A function* $f : X \to Y$ *is almost* γ*-continuous if and only if* $\tau_\gamma\text{-}Cl(f^{-1}(sInt(F)))) \subseteq f^{-1}(F)$ *for each preclosed set* F *of* Y*.*

Theorem 9. *For a function* $f : X \to Y$*, the following statements are equivalent:*

(i) f *is almost* γ*-continuous.*

(ii) For each neighborhood V *of* $f(x)$*,* $x \in \tau_\gamma\text{-}Int(f^{-1}(sCl(V)))$*.*

(iii) For each neighborhood V *of* $f(x)$*,* $x \in \tau_\gamma\text{-}Int(f^{-1}(Int(Cl(V))))$*.*

Proof. Follows from Theorem 8 and Corollary 2.

Theorem 10. *Let* $f : X \to Y$ *is an almost* γ*-continuous function and Let* V *be any open subset of* Y*. If* $x \in \tau_\gamma\text{-}Cl(f^{-1}(V)) \setminus f^{-1}(V)$*, then* $f(x) \in \tau_\gamma\text{-}Cl(V)$*.*

Proof. Let $x \in X$ be such that $x \in \tau_\gamma\text{-}Cl(f^{-1}(V)) \setminus f^{-1}(V)$ and suppose $f(x) \notin \tau_\gamma\text{-}Cl(V)$. Then there exists a γ-open set H containing $f(x)$ such that $H \cap V = \phi$. Then $Cl(H) \cap V = \phi$ implies $Int(Cl(H)) \cap V = \phi$ and $Int(Cl(H))$ is regular open set. Since f is almost γ-continuous, by Theorem 3, there exists a γ-open set U in X containing x such that $f(U) \subseteq Int(Cl(H))$. Therefore, $f(U) \cap V = \phi$. However, since $x \in \tau_\gamma\text{-}Cl(f^{-1}(V))$, $U \cap f^{-1}(V) \neq \phi$ for every γ-open set U in X containing x, so that $f(U) \cap V \neq \phi$. We have a contradiction. It follows that $f(x) \in \tau_\gamma\text{-}Cl(V)$.

Theorem 11. *If $f : X \to Y$ is almost γ-continuous and $g : Y \to Z$ is continuous and open. Then the composition function $gof : X \to Z$ is almost γ-continuous.*

Proof. Let $x \in X$ and W be an open set of Z containing $g(f(x))$. Since g is continuous, $g^{-1}(W)$ is an open set of Y containing $f(x)$. Since f is almost γ-continuous, there exists a γ-open set U of X containing x such that$f(U) \subseteq Int(Cl(g^{-1}(W)))$. Also, since g is continuous, then we obtain $(gof)(U) \subseteq g(Int(g^{-1}(Cl(W))))$. Since g is open, we obtain $(gof)(U) \subseteq Int(Cl(W))$. Therefore, gof is almost γ-continuous.

Theorem 12. *If $f : X \to Y$ is an almost γ-continuous function and Y is semi-regular. Then f is γ-continuous.*

Proof. Let $x \in X$ and Let V be any open set of Y containing $f(x)$. By the semi-regularity of Y, there exists a regular open set G of Y such that $f(x) \in G \subseteq V$. Since f is almost γ-continuous. By Theorem 3, there exists a γ-open set U of X containing x such that $f(U) \subseteq G \subseteq V$. Therefore, f is γ-continuous.

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