EUROPEAN JOURNAL OF MATHEMATICAL SCIENCES
Vol. 2, No. 3, 2013, 352-360
ISSN 2147-5512 – www.ejmathsci.com

Applying the 0-1 Test for the Detection of Chaotic and Reduced-order Synchronization of Chaotic Systems with Different Dimension

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Abstract. In this paper, we are applying the 0-1 test for detection of chaotic of dynamical systems. Also, my purpose is synchronization of the chaotic systems with an arbitrary different order system, by applying an appropriate control signal, so that it follow behavior a chaotic system that parameters are unknown. The main feature of the reduced-order synchronization is that the order of the slave system is less than the master system. A widely variety of approaches have been proposed for chaos synchronization, such as: adaptive control, linear and nonlinear feedback control, active control.
In this paper, we after detection chaos of two chen and lu systems applying of the 0-1 test , synchronize they by the reduced-order method. Also, by choosing appropriate positive definite Lyapunov function, the stability of the system were investigated for the various parameters. The advantage of this controller design is the time variable its coefficients.

2010 Mathematics Subject Classifications: 34Hxx , 34H10

Key Words and Phrases: Synchrozation, Chaos, Control, 0-1 test, Dynamical systems, Reduced-order, Lyapunov function, Positivedefinite

1. Introduction

Chaos behavior occurs widely in natural and engineering systems, such as Chen and Lu systems. In order to investigate the reduced-order synchronization behavior between hyper-chaotic Chen system and Lu system [1], Figures 1a and 2b. In synchronization of chaotic systems proposed by L.M. Pecora and T.L. Carol [7], assumed that systems are same order while in applied problem are not same order. Reduce-order synchronization is the problem of synchronizing a slave system with projection of a master system [2]. Also in this paper, after introducing the system "Chen" and "Lu" briefly, describe show to apply the 0-1 test [7],

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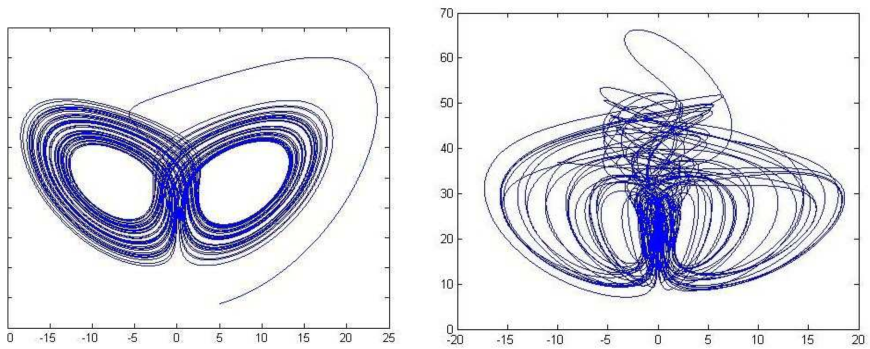
then this test will run on these systems also. In addition, how to synchronize chaotic systems non dimension, and we will explain a controller design method of active control. The hyper chaotic Chen system is given by

$$\begin{cases} \dot{x} = a(y-x)+w \\ \dot{y} = dx - xz + cy \\ \dot{z} = xy - bz \\ \dot{w} = yz + rw \end{cases} \tag{1}$$

where x, y, z and w are state variables and a, b, c, d and r, are real constant. Although the Lu system is describe by

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = -xz + cy \\ \dot{z} = xy - bz \end{cases} \tag{2}$$

where x, y, z are state variables and a, b, c are positive constant.



(a) Chen (b) Lu

Figure 1: Chen and Lu Hyperchaotic Systems.

2. Description the 0-1 Testing and Synchronization of Chaotic Systems

2.1. 0-1 Test

Before you begin to format your paper, first write and since in real applications may obtain the complete system model is costly or impossible, this test is designed even with the uncertain dynamics of the system can be chaotic at arbitrary intervals evaluated and identify all parts of the chaotic and nonchaotic. Another advantage of this method is that the test system runs directly on the time series. The introduction of such tests in the new season is in chaos theory and the method is based on the introduction of a chaotic system for sustainable development unstable periodic paths in the absorber system is chaotic. This test was presented in 2003 by Mellbourne Gatvald [4, 5]. This test is used to detect turbulence or nonchaotic dynamical system. Also directly on the time series of the main system is running and if the system is chaotic output value is number 1 and else is near 0, Figure 2. Impose a solution system dynamic is $x(t) = (x_1(t), x_2(t), x_3(t), \cdots x_n(t))$ also $\phi(x(t))$ be an arbitrary observed function of the system such as $\phi(x) = x_1 + x_2$. Choosing an arbitrary positive constant c, we define

$$\begin{cases} \theta(t) = ct + \int_0^t \phi(x(s))ds \\ p(t) = \int_0^t \phi(x(s))\cos(\theta(s))ds \\ q(t) = \int_0^t \phi(x(s))\sin(\theta(s))ds \end{cases} \tag{3}$$

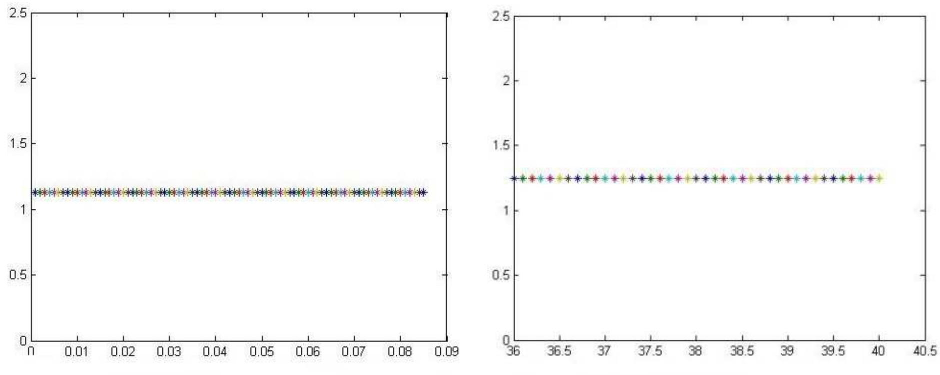
Now, for detection chaotic of system determine growth of the functions $P(t)$ and $q(t)$. Therefore, we define function $M(T)$ as follows

$$M(T) = \lim_{T\to\infty} \frac{1}{T}\int_0^T ((p(t+\tau) - p(\tau))^2 + (q(t+\tau) - q(\tau))^2)d\tau$$

Also, asymptotic growth rate of function $M(T)$ are defined as follows

$$K = \lim_{t\to\infty} \frac{\log(M(t))}{\log(t)} \tag{4}$$

Now, If K is close to zero, the system is not chaotic, and if is near one, the system is chaotic.



(a) Chen (b) Lu

Figure 2: Performance of the 0-1 Test for Detection of Chaotic Chen and Lu Systems.

2.2. Synchronization of Chaotic Systems

Generally, the synchronization of chaos is process in which two or more identical or non-identical chaotic systems, a distinct feature of motion set through a foreign force to achieve a set of common behaviors. The most common configuration for synchronous systems is considered as two subsystems are coupled, the one as slave system and the other as the master system. In fact, purpose of synchronous this is that slave system follow dynamics of master system. Chaotic systems, according to the type and intensity of the coupling between the two systems is established, with different methods are synchronous with each other [7]. For clarification, the following relationship between slave and master systems, respectively, as we think:

$$\begin{cases} \dot{u}(t) = f_u(u,t) \\ \dot{v}(t) = f_v(u,t) \end{cases} \tag{5}$$

where $u \in R^n$, $v \in R^n$ the system state variables f_u, and f_v are chaotic nonlinear functions. These two systems can, by applying an appropriate control signal, and the special relationship between their trajectories, be synchronous. In this case we have

$$lim_{t\to\infty}\|D_1 u(t) - D_1 u(t)\| \to 0 \tag{6}$$

Functions D_1 and D_2 determine the optimum relationship between state variables systems. In fact, they specify the type of synchronization

3. Active Control Method and Designing of Controllers for Synchronous Chaotic Systems with Different Dimensions

Purpose of this section is that, by applying an appropriate control signal to the chaotic system, it behaves like a chaotic system synchronized with the higher dimension. The first system that not fully known and is uncertainty and higher dimension, the master system and the second system, the control signal is entered, is call slave system [1, 6]. Note that, the control signal, enters into the second chaotic system, and the master system does not have any effect, it one-way synchronization is called. Although, dimension of slave system is less than, dimension of the master system, synchronization can only be done reduced-order. This means that the dynamic of the slave system is synchronous with the image focus of the dynamics of master system. In fact, in the reduced-order synchronization process, all state variables of slave system are synchronize. To achieve this goal, the control algorithm based on active control methods designed, that using it, making it possible to zero asymptotic. Synchronization error exists. Overview of the slave and master systems, respectively, is shown below

$$\begin{cases} \dot{x}(t) = f(x,t) + F(x,t)p \\ \dot{y}(t) = g(x,t) \end{cases} \tag{7}$$

In this relationship t Represents the time $x \in R^n$ State vector of the master system $y \in R^m, (m < n)$. State vector of the slave system $f : R^n \times R^+ \to R^n$ and $g : R^m \times R^+ \to R^m$ and $F : R^n \times R^+ \to R^{n\times k}$ are nonlinear functions and $y \in R^m$ Vector of unknown parameters of the system. In this section, making the synchronization problem to problem of stabilizing the asymptotic dynamics of synchronization error, we turn around the origin. So we consider first the master system into two parts. In the first step, the master system projection on Figure 3 that is the page codimension with slave system.

$$\begin{cases} \dot{x}_{s(t)} = f_s(x,t) + F_s(x,t)p \\ \dot{x}_{r(t)} = f_r(x,t) + F_r(x,t)p \end{cases} \tag{8}$$

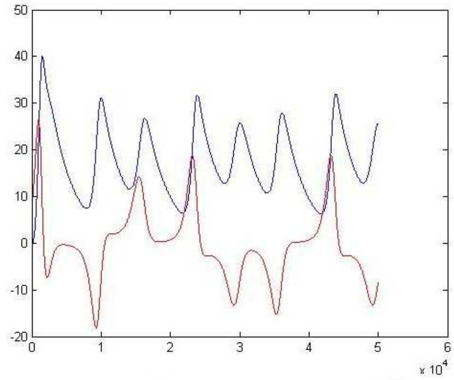


Figure 3: Performance of the 0-1 Test for Detection of Chaotic Chen System.

The nonlinear function f_s shows dynamic image on Figure 3 and f_r Describe the dynamics is The remaining part. Therefore

$$\dot{y}(t) = g(x,t) + u(t)$$

where $u \in R^n$.

$$\dot{e} = f_s(x,t) + F_s(x,t)p - g(y,t) - u$$

As we know, here the purpose control is stabilize asymptotic dynamics of error around the origin. Thus, the control signal U so we can determine that Beginning with the removal of nonlinear parts, error dynamics Become a dynamic linear.

$$u = f_s(x,t)\hat{p} + F_s(x,t)\hat{p} - g(y,t) + ke \tag{9}$$

Since P is a vector of parameters that their values are not known, Only p is estimated that, in the relationship U is considered. So after applying the control signal U and considering the certainty equivalence principle, Error equation of the form $\dot{e} = e$ is that a linear dynamic and sustainable.

4. Synchronization Between the Projection x-y-z of the Hyperchaotic Chen System and Lu System

In order to investigate the reduced-order Synchronization behavior between hyperchaotic Chen system and Lu system, we assume that the x-y-z projection of the Chen hyperchaotic systems is the master system which will be denoted by subscript 1, and we assume that the Lu system is the response system which will be denoted by subscript 2, therefore , the master system is the projection part of Equation Chen [2, 3, 8, 9].

$$X_m : \begin{cases} \dot{x}_1 = a_1(y_1 - x_1) + \omega_1 \\ \dot{y}_1 = d_1 x_1 - x_1 z_1 + c_1 y_1 \\ \dot{z}_1 = x_1 y_1 - b_1 z_1 \end{cases} \tag{10}$$

$$X_s : \begin{cases} \dot{x}_2 = a_2(y_2 - x_2) + u_1 \\ \dot{y}_2 = -x_2 z_2 + c_2 y_2 + u_2 \\ \dot{z}_2 = x_2 y_2 - b_2 z_2 + u_3 \end{cases} \tag{11}$$

Where u_1, u_2, u_3 are three control functions to be designed and all parameters a_1, d_1, b_1, c_1, a_2, b_2, c_2 are unknown constant parameters. By $E = X_m - X_s$ have

$$\begin{cases} \dot{e}_1 = \dot{x}_1 - \dot{x}_2 = a_1(y_1 - x_1) + w_1 - a_2(y_2 - x_2) - u_1 \\ \dot{e}_2 = \dot{y}_1 - \dot{y}_2 = d_1 x_1 - x_1 z_1 + c_1 y_1 + x_2 z_2 - c_2 y_2 - u_2 \\ \dot{e}_3 = \dot{z}_1 - \dot{z}_2 = x_1 y_1 - b_1 z_1 - x_2 y_2 + b_2 z_2 - u_3 \end{cases} \tag{12}$$

where

$$\begin{cases} e_1 = x_1 - x_2 \\ e_2 = y_1 - y_2 \\ e_3 = z_1 - z_2 \end{cases} \tag{13}$$

Our goal is to find proper control functions u_1, u_2, u_3, and parameter update rule, such that system Equation (11) asymptotically synchronizes system Equation (10) i.e. Figures 4-6.

$$lim_{t\to\infty} \parallel e(t) \parallel = 0 \tag{14}$$

Where $e = (e_1, e_2, e_3)$, [1, 7, 9]. For this end, we propose the following adaptive control laws for system Equation (11).

$$\begin{cases} u_1 = \hat{a}_1(y_1 - x_1) + w_1 - \hat{a}_2(y_2 - x_2) + ke_1 \\ u_2 = \hat{d}_1 x_1 - x_1 z_1 + \hat{c}_1 y_1 + x_2 z_2 - \hat{c}_2 y_2 + ke_2 \\ u_3 = x_1 y_1 - \hat{b}_1 z_1 - x_2 y_2 + \hat{b}_2 z_2 + ke_3 \end{cases} \tag{15}$$

And parameter update rules

$$\begin{cases} \dot{\hat{a}}_1 = -(y_1 - x_1)e_1, \\ \dot{\hat{b}}_1 = -z_1 e_3, \\ \dot{\hat{d}}_1 = -x_1 e_2, \\ \dot{\hat{c}}_1 = -y_1 e_2, \\ \dot{\hat{a}}_2 = -(y_2 - x_2)e_1, \\ \dot{\hat{b}}_2 = -z_2 e_3, \\ \dot{\hat{c}}_2 = -y_2 e_2. \end{cases} \tag{16}$$

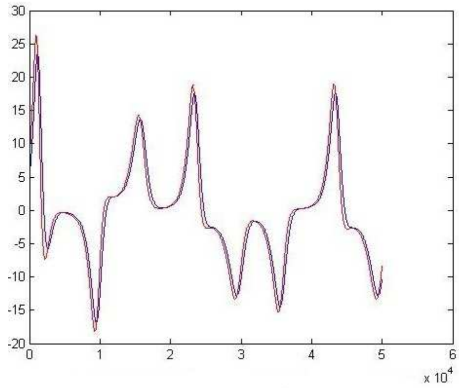


Figure 4: Performance of the 0-1 Test for Detection of Chaotic Chen System.

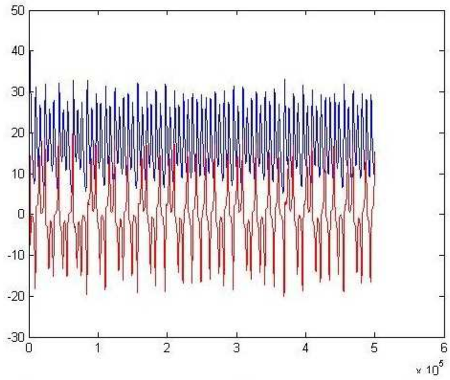


Figure 5: Performance of the 0-1 Test for Detection of Chaotic Chen System.

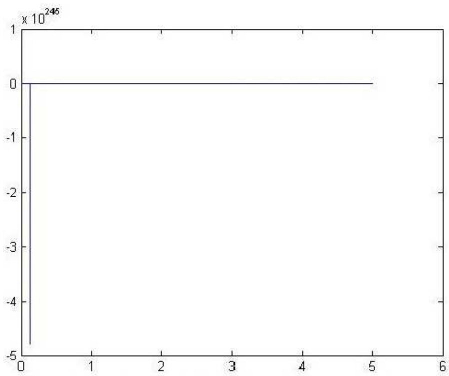


Figure 6: Performance of the 0-1 Test for Detection of Chaotic Chen System.

Theorem 1. *For any initial conditions the Lu system is synchronize with the x-y-z projection of hyper chaotic Chen system asymptotically synchronize by using adaptive control laws Equation* (15) *and parameter update rules Equation* (16).

Proof. Have

$$\begin{aligned}\dot{e}_1 =& \tilde{a}_1(y_1 - x_1) + \tilde{a}_2(y_2 - x_2) - e_1\\ \dot{e}_2 =& \tilde{d}_1 x_1 + \tilde{c}_1 y_1 + \tilde{c}_2 y_2 - e_2\\ \dot{e}_3 =& -\tilde{b}_1 z_1 - \tilde{b}_2 z_2 - e_3\end{aligned}$$

where $\tilde{a}_1 = a_1 - \hat{a}_1$, $\tilde{b}_1 = b_1 - \hat{b}_1$, $\tilde{c}_1 = c_1 - \hat{c}_1$, $\tilde{d}_1 = d_1 - \hat{d}_1$, $\tilde{a}_2 = a_2 - \hat{a}_2$, $\tilde{b}_2 = b_2 - \hat{b}_2$, $\tilde{c}_2 = c_2 - \hat{c}_2$. Consider the Lyapunov function candidate

$$v(t) = \frac{1}{2}\sum_{i=1}^{3} e_i^3(t) + \frac{1}{4}\sum_{i,j=1}^{6} \delta_{i,j}^2(t) \tag{17}$$

therefore

$$v(t) = \frac{1}{2}(e_1^2(t) + e_2^2(t) + e_3^2(t) + \tilde{a}_1^2 + \tilde{b}_1^2 + \tilde{d}_1^2 + \tilde{c}_1^2 + \tilde{a}_2^2 + \tilde{b}_2^2 + \tilde{c}_2^2) > 0$$

$$\dot{v}(t) = (e_1^2(t) + e_2^2(t) + e_3^2(t)) \leq 0$$

Since v is positive definite and $\dot{v}$ is negative definite therefore

$$lim_{t\to\infty}|e_i(t)| = 0, \quad i = 1, 2, 3$$

□

5. Conclusion

In this paper, algorithms for building reduced-order synchronization of uncertain chaotic systems was presented The overall configuration of the system is as a slave-master, that dimension of slave system is consider lower than dimension of master system. Therefore, the constraint co-dimension systems, which in many ways been seen in the synchronization of chaos, no. and slave system is synchronous with Picture of the master system and The controller design active control method has been made. Approach, the asymptotic error to zero in the presence of uncertain parameters. In this paper, the 0-1 test for the detection of chaotic systems is used.and Also, by choosing appropriate positive definite Lyapunov function, the stability of the system investigated for the various parameters.

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